CALCULATIONS ON THE ENTROPY-TEMPERATURE CHART.

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PREFACE.

My object in writing this little book is to show students who are not very well up in thermodynamics how to make use of the entropy-temperature chart. For this purpose I have treated the matter as simply as possible, and have not insisted on a first-rate knowledge of the principles underlying entropy as the indispensable outfit for the purpose in hand. There are some subjects in which a teacher is justified in withholding information if he thinks that information is practically incapable of being assimilated by a student—or, at the most, being imperfectly assimilated—and I submit that entropy is one of these. It is much better to teach a student how to make calculations on the chart than to be eternally drilling him in the abstract mathematical principles involved; in the case of the advanced student, the latter will sooner or later force themselves automatically upon his attention, and in the case of evening students and of others whose mathematical training is of a low standard the former is the important thing, even if the latter be never properly appreciated.

None of us like to be told how to do a thing without learning the ins and outs of that thing; and, in all cases where those ins and outs can be fairly easily understood, much the best policy is to teach them first. Nevertheless, the world would be a peculiar place if everybody insisted on knowing the why and wherefore of everything. If we cannot understand the origin of a thing we must take it as we find it, make use of it, and hope for its solution later. It would be absurd to refuse to teach the use of a spade to a child because he does not understand all the processes of its manufacture.

My hope is that this little volume will be of use to those who would like to learn how to use the chart but who have never been able to find the information in a sufficiently simple form. Therefore, I have introduced a large number of examples, and have worked many of these out at length. There are simple examples and there are difficult ones, and the student must pick and choose for himself; and whether he is an elementary or advanced student, he must have the chart before him and work out the problems for himself and obtain numerical results.

The Board of Education issues a splendid entropy-temperature chart at the price of one penny. This can be purchased, either directly or through any bookseller, from Wyman & Sons, Ltd., Fetter Lane, London, E.C.; from Oliver & Boyd, Tweeddale Court, Edinburgh; or from E. Ponsonby, Ltd., 116 Grafton Street, Dublin. The chart is free from any superfluous matter, and calculations can be easily and accurately made upon it. I advise all students of the subject to obtain it at once.

In the preparation of the volume I have been indebted to most of the Standard Works on Thermodynamics, but chiefly those of Professors Perry, Ewing, and Ripper. The Board of Education Examination Questions are published by permission of the Controller of His Majesty's Stationery Office and those of the Institute of Civil Engineers by permission of the printers.

In conclusion, I shall be glad to hear from teachers and students any suggestions for the improvement of the subject-matter or notifications of any errors in the text or examples.

W. J. CRAWFORD.

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CHAPTER I.

DEFINITIONS, EXPLANATIONS, AND EXAMPLES.

It is necessary that the student should be thoroughly familiar with the following, before proceeding further with the study of the subject :—

(1) The Centigrade Heat Unit (C.H.U.) is the heat necessary to raise 1 lb. of water through 1° C.

(2) The Fahrenheit Heat Unit (F.H.U.), also often called the British Thermal Unit, is the heat necessary to raise 1 lb. of water through 1° F.

(3) The number of heat units given to any substance is the product of three quantities, the *weight in pounds*, the *specific heat*, and the *rise in temperature.*

(4) The specific heat of water is approximately unity, hence when finding the quantity of heat given to water, we simply multiply together the weight of the water in pounds and the rise of temperature.

(5) The *Absolute* zero of temperature is 273° below "zero Centigrade," and 461° below "zero Fahrenheit."

(6) When water is heated in a boiler, its temperature is first raised to the boiling point, and it is then converted into steam at that temperature.

(7) The heat required to raise 1 lb. of the water to the boiling temperature, is called the *Sensible heat.*

(8) The heat required to convert 1 lb. of water at the boiling temperature into 1 lb. of steam at the same temperature, is called the *Latent heat.*

The sum of the sensible and latent heats is called the *Total heat.*

The latent heat varies with the temperature at which the water is converted into steam. It can be found in the steam tables for all temperatures used in practice.

If the tables are not handy, the latent heat can be found from formulæ, known as *Regnault's total heat formulæ.* They are :—

$$H = 606{\cdot}5 + {\cdot}305\, t^\circ \text{ (for Centigrade temperatures)},$$
$$H = 1082 + {\cdot}305\, t^\circ \text{ (for Fahrenheit temperatures)},$$

where H is the total heat necessary to raise 1 lb. of water from the freezing temperature (0° C. or 32° F.), up to the boiling temperature t°, *whatever that may be,* and to convert it into 1 lb. of dry steam at that temperature.

It must be clearly understood that, when calculating the total heat by Regnault's formulæ, the pound of water is always at the freezing point to begin with.

To find, by Regnault's total heat formula, the latent heat of steam at 160° C.

$$\begin{aligned} H &= 606{\cdot}5 + {\cdot}305\, t^\circ \\ &= 606{\cdot}5 + {\cdot}305 \times 160 \\ &= 606{\cdot}5 + 48{\cdot}8 \\ &= 655{\cdot}3 \text{ C.H.U.} \end{aligned}$$

That is, the total heat required to raise 1 lb. of water from 0° C. to 160° C. and then to convert it into steam at this temperature is 655·3 C.H.U. Now, this total heat consists of the sum of two quantities, the sensible heat and the latent heat. Hence, if we subtract the sensible heat from the total heat we obtain the latent heat. The sensible heat is the heat necessary to raise the pound of water from 0° C. to 160° C., and it is, therefore, the weight × rise of temperature = 1 × 160 = 160 C.H.U. The latent heat is then total heat — sensible heat = 655·3 — 160 = 495·3 C.H.U.

To find by Regnault's formula the latent heat of steam at 300° F.

We have,
$$\begin{aligned} H &= 1082 + {\cdot}305\, t^\circ \\ &= 1082 + {\cdot}305 \times 300 \\ &= 1082 + 91{\cdot}5 \\ &= 1173{\cdot}5 \text{ F.H.U.} \end{aligned}$$

That is to say, the heat necessary to raise 1 lb. of water from 32° F. to 300° F., and to convert it into steam at that temperature, is 1173·5 F.H.U. To obtain the latent heat we must subtract the sensible heat, the latter being the heat necessary to raise the 1 lb. of water from 32° F. to 300° F. $= 1 \times (300 - 32)$

$$= 1 \times 268$$
$$= 268 \text{ F.H.U.}$$

The latent heat is, therefore, $1173{\cdot}5 - 268 = 905{\cdot}5$ F.H.U.

To find the total heat necessary to raise 1 lb. of water from a feed temperature of 30° C. to a boiling temperature of 160° C., and to convert it into steam at that temperature.

We must add together the sensible and latent heats. The sensible heat is $1 \times (160 - 30) = 130$ C.H.U. The latent heat we find by Regnault's formula, as explained above, to be 495·3 C.H.U. The total heat required is then $130 + 495{\cdot}3 = 625{\cdot}3$ C.H.U.

The latent heat can be divided into two portions, the *external* heat and the *internal* heat.

The external portion of the latent heat is the heat necessary to enlarge the volume of the 1 lb. of water into 1 lb. of steam, against the superincumbent pressure. Suppose we have a boiler half full of water with steam filling the steam space. When 1 lb. of the water is converted into 1 lb. of steam, it must, in order to make a space for itself, push away some of the steam above it, and the heat (or work) necessary to do this is the external portion of the latent heat.

$$\text{The external heat} = \frac{P\,V}{J},$$

where P = pressure of steam in pounds per square foot.
V = volume in cubic feet of 1 lb. of the steam.
J = Joule's equivalent, or foot-pounds of work in 1 heat unit (1,393 for 1 C.H.U., and 774 for 1 F.H.U.).

The internal heat is the heat necessary to break up the molecules of water and convert them into free steam molecules. The internal heat is much the greater portion of the latent heat.

The *intrinsic* or *internal* energy possessed by 1 lb. of steam is its total heat minus the external portion of its latent heat.

Saturated steam is steam at the temperature of its formation, and is usually steam in contact with the water from which it is formed. Saturated steam is not necessarily wet steam, as its name would seem to imply, but may be perfectly dry.

Wet steam is steam that has some water particles suspended in it.

Superheated steam is steam that is heated after its formation.

If steam is wet, the total heat necessary to raise 1 lb. of feed water to the boiling temperature, and to convert it into 1 lb. of wet steam at that temperature is :—

$$\text{Total heat} = s + q\,L,$$

where s is the sensible heat, L is the latent heat, and q is the *dryness* fraction.

Expansion of Gases.—*Boyle's law* states that if the temperature remains constant, the pressure of a gas varies inversely as its volume *or* its volume varies inversely as the pressure.

That is, $$P \propto \frac{1}{V}; \text{ or, } P\,V = k,$$

where P is the pressure, V is the volume, and k is a constant.

Charles' law states that if the pressure remains constant the volume of a gas varies directly as the absolute temperature.

That is, $$V \propto T; \text{ or, } V = m\,T,$$

where V is the volume, T is the absolute temperature, and m is a constant.

If the temperature, pressure, and volume of a gas all vary, we may combine the above two laws, thus—

$$P\,V = R\,T,$$

where P = pressure, V = volume, T = absolute temperature, R = a constant.

A gas may expand in many ways, but the two chief are *Isothermal* and *Adiabatic* expansions.

Isothermal expansion is expansion at constant temperature. As the gas expands and loses heat, heat must be continually given it from an outside source, in order to keep its temperature constant.

Adiabatic expansion is such that no heat leaves, or is given to, the gas during the expansion, to or from an outside source.

Steam is not a perfect gas, because there is usually water suspended in it. Hence in steam engines we have usually a mixture of steam and water expanding. Isothermal expansion is approximately obtained when a steam jacket surrounds the cylinder in order to keep the expanding mixture at as even a temperature as possible. We have approximately adiabatic expansion when the cylinder is thoroughly lagged with non-conducting material, so that very little heat may get into or leave the cylinder.

We shall in the succeeding pages have a great deal to do with these two kinds of expansion, and it is very necessary to understand clearly what they mean.

The Indicator Diagram.—The student is assumed to be familiar with this. He must know that its area represents work, how to obtain the mean height of the diagram and hence the mean effective pressure, how to calculate the indicated horse-power, and so on. He must also understand the main portions of the diagram, such as admission, cut-off, release, and compression. He must know the meaning of clearance volume, absolute and gauge pressures, atmospheric line, etc. In other words, he should have a very good working knowledge of the indicator diagram.

$$\text{Efficiency} = \frac{\text{work got out}}{\text{work put in}} = \frac{\text{heat got out}}{\text{heat put in}}.$$

$$\left.\begin{array}{l}\text{The } \textit{mechanical} \text{ efficiency} \\ \quad\text{of an engine}\end{array}\right\} = \frac{\text{brake horse-power}}{\text{indicated horse-power}}.$$

$$\left.\begin{array}{l}\text{The } \textit{thermal} \text{ efficiency of} \\ \quad\text{an engine}\end{array}\right\} = \frac{\text{heat converted into useful work}}{\text{total heat expended}}.$$

There are two hypothetically *perfect* engines with which we can compare the working of ordinary engines. These are the Carnot and Rankine engines respectively. Full particulars of these are given in the succeeding pages.

CHAPTER II.

TO DRAW THE WATER AND STEAM LINES ON THE ENTROPY CHART.

Entropy.—Entropy is the sum of all the small quantities of heat given to a body, each divided by the absolute temperature at which the heat is given.

The meaning and significance of the term "entropy" are somewhat difficult to understand at first, and the student need not despair if he does not fully comprehend them at the commencement of his task. He will, at anyrate, be soon able to understand the chief points on the entropy-temperature chart, and to work calculations upon it, and gradually the meaning of entropy will so force itself upon him, that in the end he will grasp it thoroughly.

Suppose we have a pound weight of water at, say, 30° C.; we give this water a small quantity of heat, say, 2 C.H.U. These two units will raise its temperature to 32° C.—*i.e.*, from 303° C. absolute to 305° C. absolute—then the average *absolute temperature* at which we have given our two units of heat to the pound of water is $\frac{303 + 305}{2} = 304°$ C. absolute. Hence, as entropy is the small amount of heat given to the water divided by the absolute temperature at which it is given, the entropy we have added to the water is $\frac{2}{304} = \cdot 00655$ unit.

Now, suppose we give another small increment of heat to the water, say another 2 C.H.U. This will raise the temperature a further 2° C.—namely, from 305° C. absolute to 307° C. absolute. Therefore, the average absolute temperature at which the new increment of heat is added to the water is $\frac{305 + 307}{2} = 306°$ C. absolute. The increase of entropy due to this is then $\frac{2}{306} =$

·00653 unit; and so on. Finally, the sum of all the increments of heat given to the water, each divided by the mean or average absolute temperature at which it is given, is the total amount of entropy added to the water.

The unit of entropy is called the "*rank.*" In order to indicate graphically the various quantities of heat given to water and steam, and to show the various amounts of heat converted into work by expanding steam, and to show many other thermal changes which could only otherwise be represented by long and fatiguing mathematical processes, an entropy-temperature chart can be constructed. On this chart entropy is plotted horizontally and temperature vertically. The entropy-temperature chart is usually abbreviated into the "θ-φ" chart (the "theta-phi" chart), the Greek symbols θ and φ being used for absolute temperature and entropy respectively.

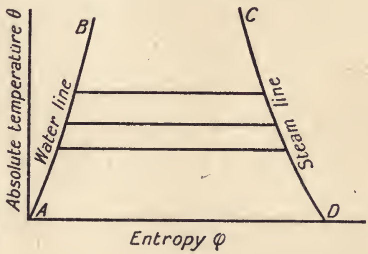

Fig. 1.

When we look at such a chart, we see that there are two main portions to it, the curves A B and C D (fig. 1); connecting these two curves there is a system of horizontal lines.

The curve A B is called the "water line" or the "water curve," and the curve C D is known as the "saturated steam curve."

It is now necessary to show how the water and saturated steam curves are drawn. The student will have the B. of E. or similar chart before him, upon which these lines are already drawn; nevertheless, he is advised to obtain a large piece of squared paper and to construct the curves for himself.

To construct the water line on the θ-φ chart.

Note.—The zero of entropy is always taken at 0° C. or 32° F. (ordinary).

To construct the water line from first principles, we must keep constantly in mind the definition of entropy—namely, that it is the *small increment of heat* given to the water *divided by the mean absolute temperature* at which it is given.

On the θ-φ chart we deal with 1 lb. weight of water, and this fact must be remembered always.

The zero of entropy being at 32° F., we take 1 lb. of water at that temperature. Suppose we give it a small increment of heat, say 18 F.H.U.

The quantity of heat given to any body is the product of three things, the *weight in lbs.*, the *specific heat*, and *the rise in temperature.*

Therefore, 18 = 1 lb. × specific heat of water × rise in temperature.

The specific heat of water is nearly constant at unity. At high temperatures the specific heat is a little greater than unity, but the difference is so slight, that we shall be making no appreciable error if we consider it unity throughout our series of calculations.

We have, then, 18 = 1 × 1 × rise of temperature. Therefore, the rise of temperature is 18° F. So it comes to this, that if we give the 1 lb. of water any number of heat units, the rise of temperature is the *same* number of degrees F.

The rise in temperature is 18° F.—that is, from 32° F. to 50° F. But absolute zero F. is 461° F. below zero F. (ordinary). Hence 32° F. = 493° F. absolute, and 50° F. = 461 + 50 = 511° F. absolute. Hence the rise of temperature is from 493° F. absolute to 511° F. absolute. Therefore, the absolute *average* temperature at which the 18 F.H.U. are given to the water is $\frac{493 + 511}{2} = \frac{1{,}004}{2} = 502°$ F. absolute.

$$\left.\begin{matrix}\text{Increase of}\\ \text{entropy}\end{matrix}\right\} = \frac{\text{small quantity of heat given water}}{\text{average absolute temperature at which heat is given}}$$

$$= \frac{18}{502}$$

$$= \cdot 0358 \text{ ranks.}$$

Or, in words:—If we have 1 lb. of water at 32° F., and give it a small quantity of heat—namely, 18 F.H.U.—its entropy is increased by ·0358 ranks. As the entropy the water possesses at 32° F. is reckoned as zero, the *total* entropy the water possesses because of the 18 F.H.U. we have given it is ·0358 ranks. Hence, on the θ-φ chart, plot ·0358 unit of entropy against 50° F.

Our 1 lb. of water is now at 50° F. ordinary, or 511° F. absolute. Let us give it another small increment of heat, say 20 F.H.U. As explained above, its temperature will rise 20° F. further—that is, from 511° F. to 531° F. absolute. Then the average absolute temperature at which the new increment of heat has been given to the water is $\frac{511 + 531}{2} = \frac{1{,}042}{2} = 521°$ F.

$$\text{Increase of entropy} = \frac{\text{increment of heat}}{\text{average absolute temperature}}$$

$$= \frac{20}{521}$$

$$= \cdot 0384 \text{ ranks.}$$

This is the increase of entropy above what the water had at 50° F. ordinary. We found that it had then ·0358 ranks, so that the *total increase of entropy* from 32° F. (where the entropy is zero) is ·0358 + ·0384 = ·0742 ranks. On the θ-φ chart plot ·0742 ranks against 70° F.

Again, let us give our 1 lb. of water, now at 70° F., another increment of heat, say 20 F.H.U. more. This will raise its temperature a further 20° F.—that is, from 531° F. absolute to 551° F. absolute. Then the average absolute temperature at which the further increment of heat has been given to the water is $\frac{531 + 551}{2} = \frac{1{,}082}{2} = 541°$ F.

$$\text{Increase of entropy} = \frac{\text{increment of heat}}{\text{average absolute temperature}}$$

$$= \frac{20}{541}$$

$$= \cdot 0370 \text{ ranks.}$$

This is the increase of entropy above what the 1 lb. of water had at 70° F.—namely, ·0742 ranks. Therefore, the total entropy of the water from 32° F. is ·0742 + ·0370 = ·1112 ranks.

So, on the θ-φ chart, plot ·1112 ranks against 90° F.

Continuing in this way, giving small increments of 20 F.H.U. to the water, and dividing each by the average absolute temperature at which it is given, thus obtaining the increase of entropy due to each; adding to this the entropy the water possesses owing to previous increments of heat given to it, and plotting this total entropy against the temperature, up to a temperature of about 420° F., which is high enough for all practical purposes for which the entropy-temperature chart is used, it will be found that the water line as thus constructed is very nearly straight. Of course, the smaller the increments of heat are taken, the more accurate will be the work.

But besides this fundamental method of constructing the water line, there is an easier and more convenient method based on mathematical principles, and for the sake of becoming familiar with the elements of the chart, the water line should also be constructed in this way.

The entropy of water is given by the following formula:—

$$\varphi_w = \log_e \frac{\tau}{273}.$$

where φ_w means the entropy of water, τ means the absolute temperature Centigrade at which the increment of heat is given to the water, and 273 is the number of degrees below ordinary zero C. of absolute zero C. Note that the above formula is for calculation in C. degrees. The F. formula is the same, with the exception that 273 is replaced by 493, and τ refers to absolute degrees F.

For the sake of variety, the C. formula will be used in calculating entropies by this method. The method is extremely simple. The entropy of water at zero C. (ordinary) is considered to be nothing, just as was the case at 32° F. (= 0° C.).

To calculate the entropy of water at 10° C. = 10 + 273 = 283° C. absolute.

We have, $$\varphi_w = \log \frac{283}{273} = \log_e 1{\cdot}0366$$

$$= {\cdot}0359 \text{ ranks}.$$

This is the entropy the 1 lb. of water possesses at 10° C. above what it had at 0° C. (when its entropy was zero), so plot ·0359 ranks against 10° C.

Again, calculate the entropy of the water at 20° C. = 20 + 273 = 293° C. absolute.

We have, $\varphi_w = \log_e \frac{293}{273} = \log_e 1{\cdot}0733$

$= {\cdot}0704$ ranks.

This is the entropy the water possesses at 20° C. above what it had at 0° C. (when its entropy was zero), so plot ·0704 ranks against 20° C.

Again, calculate the entropy of the water at 30° C. = 273 + 30 = 303° C. absolute.

We have, $\varphi_w = \log_e \frac{303}{273} = \log_e 1{\cdot}1099$

$= {\cdot}1042$ ranks.

This is the entropy the water possesses at 30° C. above what it had at 0° C. (when its entropy was zero), so plot ·1042 ranks against 30° C.

Go on in this way, finding the entropy for the 1 lb. of water at intervals of each 10° C., up to about 220° C., which is quite sufficient for all practical purposes.

The water lines obtained by the two methods should be compared, and it will be seen that there is very little difference. The two temperature scales should be placed on the vertical ordinate, Fahrenheit and Centigrade, the comparison being thus easily made.

Before proceeding to construct the *saturated steam line*, consider first how much entropy must be given to a pound of water to convert it into a pound of steam. Suppose we have 1 lb. of water at 200° F. to be converted into 1 lb. of dry saturated steam at 200° F.

Bear in mind that entropy is the sum of all the small increments of heat given the substance, each divided by the mean absolute temperature at which the heat is given.

Note.—The total amount of heat we can give 1 lb. of water to convert it into dry saturated steam, is the latent heat; and for this temperature the latent heat is 974 F.H.U. approximately. Again, 200° F. is 200 + 461 = 661° F. absolute. Note also, that all the time we are converting the water into steam, the temperature does not change; it remains at 661° F. absolute.

Let us give 20 F.H.U. to the pound of water. The absolute temperature at which it is given is 661° F. Hence the increase of entropy is $\frac{20}{661} = {\cdot}0302$ ranks.

Let us give another 20 F.H.U. to our pound of water. The absolute temperature is again 661° F. Hence the increase of entropy is $\frac{20}{661} = \cdot0302$ ranks. Another 20 F.H.U. will cause a further increase of entropy of ·0302 ranks, and so on. Now, the total quantity of heat units we can give the water is 974 F.H.U., hence it is apparent that the *total entropy* given the 1 lb. of water is $\frac{20}{661} + \frac{20}{661} + \frac{20}{661} +$ so on, until all the heat units (974) are used up. That is, the total entropy is $\frac{974}{661}$; or, in symbols, $\frac{L}{\tau}$, where L is the latent heat corresponding to an absolute temperature τ. So that all we have to do to find the entropy necessary to convert 1 lb. of water at any given temperature into 1 lb. of dry saturated steam at the same temperature, is to divide the latent heat corresponding to the given temperature by the temperature expressed in absolute degrees.

In fig. 2 let A B be the water line. Let B be a point on the water line opposite 200° F. To obtain the corresponding point M on the *saturated steam line*, find the latent heat of steam at 200° F. It is 974·18 F.H.U. The absolute temperature F. corresponding to 200° F. ordinary is 200 + 461 = 661°. Then

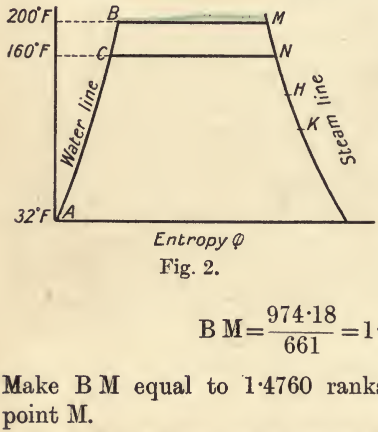

Fig. 2.

$$B\,M = \frac{974 \cdot 18}{661} = 1 \cdot 4760 \text{ ranks.}$$

Make B M equal to 1·4760 ranks to scale, and we obtain the point M.

Let C be a point on the water line opposite, say, 160° F. To obtain point N on the saturated steam line, find the latent heat of steam at 160° F. It is 1002·37 F.H.U. The absolute temperature F. corresponding to 160° F. is 160 + 461 = 621°. The entropy required is then $\frac{1002 \cdot 37}{621} = 1 \cdot 6167$ ranks.

Measure off, therefore, C N horizontally, to scale, to equal 1·6167 ranks.

Other points on the saturated steam line are obtained in this way, and when enough points have been found, the curve can be drawn in. The student should construct the curve for himself, beginning at about 420° F., and coming down to 32° F.

The two curves being constructed, the B. of E. chart should be studied for some time. It will be seen that temperatures are given in ordinary degrees to both the F. and C. scales; pressures corresponding to these temperatures are given on the right-hand ordinate, so that, when using the chart, there is no need to look up in the tables pressures corresponding to given temperatures or *vice versâ*. It should also be noted that 1 square inch of area of the chart represents 5 C.H.U., or 9 F.H.U., or 6,970 foot-lbs. Again, the absolute base is 12 inches (to scale) below a marked line on the diagram.

The following table of entropies for water and steam may be useful. In calculating the former, allowance has been made for the increased specific heat of water at high temperatures:—

Temperature. Deg. Fah.	Total Entropy above Water at 32° F.	Entropy in Forming 1 Lb. of Steam from Water.	Total Entropy of Water and Steam.
32	0	0	0
50	0·0359	2·1163	2·1522
100	0·1296	1·8649	1·9945
150	0·2154	1·6547	1·8701
200	0·2949	1·4760	1·7709
250	0·3690	1·3220	1·6910
300	0·4385	1·1880	1·6265
350	0·5042	1·0698	1·5740
400	0·5665	0·9649	1·5314

CHAPTER III.

TO MAKE SIMPLE CALCULATIONS ON THE ENTROPY-TEMPERATURE CHART.

(Chiefly on *dryness after adiabatic expansion.*)

THE water and saturated steam lines, the temperature and pressure scales, and the entropy scale having been mastered, many simple calculations can be made.

EX. 1.—A pound of water at 100° C. is heated to 150° C. and then converted into dry saturated steam at that temperature; the steam is then expanded adiabatically to 100° C. Find its dryness after the expansion.

In fig. 3, mark the point A on the water line opposite 100° C. Go up the water line to B opposite the 150° C. mark. From B go horizontally across to the saturated steam line at C. *From C draw a vertical line downwards to meet the horizontal line through A at D.* Then the dryness after the expansion is given by the ratio of $\frac{AD}{AE}$. Measure A D and A E carefully with a rule reading to small decimal parts of an inch.

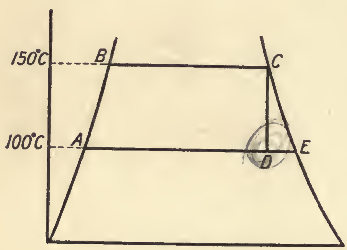

Fig. 3.

Ans. $\frac{AD}{AE} = \cdot 91$, therefore the percentage dryness is 91.

That is to say, in the 1 lb. of steam originally dry and saturated at 150° C., there is only ·91 lb. of steam after the adiabatic expansion to 100° C., and consequently ·09 lb. of water

Note.—The steam is *dry and saturated* at 150° C., so we must go to the point C. on *the saturated steam line* before beginning the expansion. The expansion is *adiabatic*, therefore we must draw a *vertical* line to represent it on the θ-ϕ chart. Remember, then, that an adiabatic line is a *line of constant entropy;* an isothermal line is a *line of constant temperature;* so that the former is always *vertical* and the latter always *horizontal* on the chart. In drawing the vertical line for adiabatic expansion, see that it is really vertical by using set squares.

EX. 2.—A pound of water is at 70° C., and is heated to 150° C.; it is then converted into steam 80 per cent. dry at that temperature, and then expanded adiabatically to 100° C. Find the dryness of the steam after the expansion.

Mark point A (fig. 4) on the water line opposite 70° C. Go up the water line to point B opposite the 150° C. mark. Go along horizontally from B till you come to C, which is $\frac{8}{10}$ the length of B K, the distance between the water and steam curves. The point C is easily found, because there are sloping lines drawn downwards on the chart at each tenth between water and steam lines. From C draw a vertical line downwards to meet the horizontal line through E, drawn opposite the 100° C. mark at D. E F is the length between the water and steam lines.

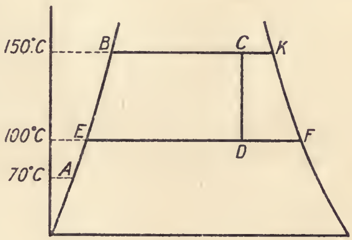

Fig. 4.

Then the dryness fraction after the expansion is given by $\frac{\text{E D}}{\text{E F}}$.

$$\textit{Ans.} \quad \frac{\text{E D}}{\text{E F}} = \cdot 75, \text{ and dryness} = 75 \text{ per cent.}$$

If it had been required to find the wetness after the expansion, we should have obtained it from the fraction $\frac{\text{D F}}{\text{E F}} = \cdot 25 = 25$ per cent.

EX. 3.—A pound of water at 150° C. is converted into dry saturated steam at that temperature. Find the amount of entropy added to it.

Mark A (fig. 5) on the water line opposite the 150° C. mark.

Go along to B horizontally on the saturated steam line.

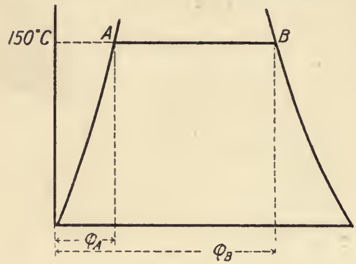

Fig. 5.

From points A and B, project down on to the entropy scale. The entropy of the water at point A is φ_A, and the entropy of the steam at point B is φ_B. Then the entropy added to the water to make it into dry saturated steam is $\varphi_B - \varphi_A$. This equals the length A B, which should be measured with the dividers and the distance marked off on the entropy scale.

Ans. 1·18 ranks.

EX. 4.—A pound of feed water at 80° C. is heated and converted into steam at a pressure of 100 lbs. per square inch. The steam is wet. After it is formed it is expanded adiabatically to a pressure of 30 lbs. per square inch, when it is found to be 60 per cent. dry. Find the original dryness of the steam.

Mark point A (fig. 6) on the water line corresponding to 80° C.

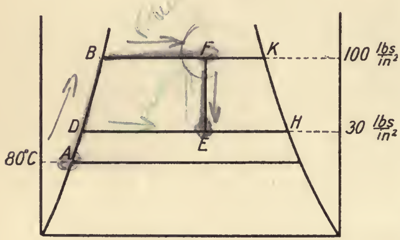

Fig. 6.

Go up the water line to B opposite the 100 $\frac{\text{lbs.}}{\text{in.}^2}$ mark. Opposite the 30 $\frac{\text{lbs.}}{\text{in.}^2}$ mark, draw a horizontal line D H between the water and steam lines. Mark off from D H, D E equal to ·6 of its length to indicate a dryness of 60 per cent. after expansion. From E draw a vertical line E F upwards to meet the top horizontal line at F. Then the original dryness of the steam formed at a pressure of 100 $\frac{\text{lbs.}}{\text{in.}^2}$ is given by $\frac{BF}{BK}$.

Ans. $\frac{BF}{BK} = \cdot 61 = 61$ per cent.

EX. 5.—A pound of water at 150° C. is converted into dry saturated steam; it is then expanded, so that it always keeps dry and saturated, to 100° C. How much entropy above what it has when dry and saturated at 150° C. must be added to it to accomplish this?

Mark point A (fig. 7) on the water line opposite 150° C. Go along horizontally to B on the saturated steam line. Go down the *saturated steam line* to D opposite 100° C. (The reason for going down the saturated steam line is that during expansion the steam has always to be dry and saturated.) Find the entropy of the steam at B, and call it φ_1; find the entropy of the steam at D, and call it φ_2. Then the increase of entropy is $\varphi_2 - \varphi_1$.

Ans. ·120 ranks.

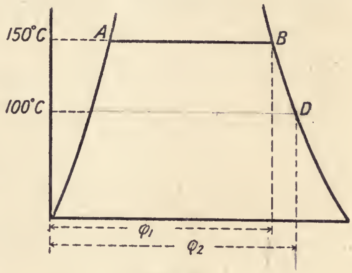

Fig. 7.

THE FOLLOWING EXAMPLES ARE SELECTIONS OF A FEW OF THE MORE ELEMENTARY QUESTIONS SET AT THE BOARD OF EDUCATION EXAMINATIONS:—

EX. 6.—To what uses do we put a θ-φ diagram? What is entropy? What is the difference between the entropies of a pound of steam and a pound of water at the same temperature?

There are many important uses of the θ-φ chart. The chief use is that all the various losses and exchanges of heat can be represented graphically. The area of the diagram represents heat units. The diagram often shows in a simple graphic way what could only otherwise be obtained by long and fatiguing mathematical processes.

Entropy is somewhat difficult to define. It is the sum of all the small increments of heat added to a body, each divided by the mean absolute temperature at which it is added.

The entropy of a pound of water at any temperature $t°$ (absolute) is—

$$\varphi_w = \log_e \frac{t°}{273},$$

where φ_w means the entropy of the water.

If, when the pound of water is at $t°$ absolute, it is converted into 1 lb. of dry saturated steam at that temperature, the additional entropy that must be added to it is $\frac{L}{t°}$, where L means the latent heat of the steam at absolute temperature $t°$.

The entropy of the steam is usually considered the sum of the entropy of the water, and the entropy required to convert the water into steam.

Thus $$\varphi_s = \varphi_w + \frac{L}{t°};$$

or, $$\varphi_s = \log_e \frac{t°}{273} + \frac{L}{t°},$$

where φ_s means entropy of the steam.

EX. 7.—A pound of water at 0° C. is heated as water to 150° C., and then converted into wet steam at the same temperature, with 20 per cent. water in it. Find its intrinsic energy and its entropy.

In fig. 8 it will be noticed that a horizontal line E D, which

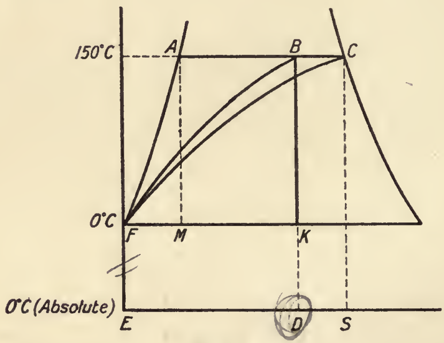

Fig. 8.

represents the absolute zero of temperature, has been drawn. This line would be much lower down than shown if it were drawn correctly to scale, but in order to save space, it has been placed as indicated on the diagram.

This example is important, and should be thoroughly mastered.

Mark point F on the water line opposite the 0° C. mark. Go up the water line to point A opposite 150° C. Draw a horizontal line A C between water and steam lines. Take point B, $\frac{8}{10}$ the length of A C. Project B downwards to the entropy scale. Then the entropy of the wet steam can be read off.

Ans. Entropy = 1·382 ranks.

From B draw a vertical line downwards to meet the line of absolute zero temperature at D. Also, from C draw a vertical line C S to the line of absolute zero temperature.

Now notice :—*If the steam be dry,* its *total heat* is represented on the entropy-temperature chart by the area E F A C S E. Or, in other words, the heat required to raise 1 lb. of water from 0° C. to 150° C., and then to convert it into dry saturated steam at that temperature is given by this area in heat units. To get total heat, we must go down to the line of absolute zero.

If the steam is wet, as at B, fig. 8, its total heat is represented on the chart by the area E F A B D E. Or, the heat necessary to raise 1 lb. of water from 0° C. to 150° C. and then to convert it into steam 80 per cent. dry, at that temperature, is given by this area in heat units.

The above results should be verified by obtaining the total heats at C and B from the tables or by calculation (see Chapter I.). The number of square inches in each of the above-mentioned areas should then be measured. For practical purposes the figure A M F may be considered as a triangle. Then, as each square inch of the chart represents 5 C.H.U., it will be seen how closely the two results agree.

Intrinsic Energy on the θ-ϕ Chart.—The following is included here because of the relation between total heat and intrinsic energy, but should be left until Chapter V. has been read.

From C and B (fig. 8) draw constant volume lines down to 0° C. Then the intrinsic energy of the dry steam at C. is the total heat minus the external latent heat, and this equals the area E F A C S E minus area F A C F (or area bounded by the water line F A, line A C, and constant-volume line C F). For practice, find the area F A C F by approximate means, and subtract this from the total heat area as found above. The result is intrinsic energy.

If the steam is wet, as at B, the intrinsic energy is the total heat minus the external latent heat, and this equals the area E F A B D E minus the area F A B F (or area bounded by the water line F A, line A B, and constant-volume line B F).

Of course, it is not recommended that the total heat and intrinsic energy should be generally found by means of the θ-ϕ chart. These quantities are much more easily found by the formulæ. Nevertheless, it is a most valuable and important exercise to thoroughly work out at least one case by both methods.

EX. 8.—Find the entropy added to 1 lb. of water at 181° C. in forming 1 lb. of wet steam at 181° C., if $\frac{9}{10}$ of it is steam and $\frac{1}{10}$ water. Sketch the appearance of the water-steam θ-ϕ diagram, and show how it informs us about liquefaction during adiabatic expansion.

Mark point A (fig. 9) on the water line opposite 181° C. Draw a horizontal line A C between the water and steam lines. Mark a point B on A C, such that A B = $\frac{9}{10}$ of A C. Then the entropy added to the 1 lb. of water at 181° C. is given by A B. Measure A B with the dividers, and read off the amount of entropy on the entropy scale.

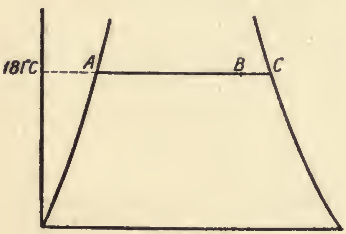

Fig. 9.

Ans. Entropy given = ·94 ranks.

Adiabatic lines are vertical lines on the θ-ϕ chart. Any such ratio as $\frac{\text{A B}}{\text{A C}}$ represents the *dryness* of steam at any particular temperature or pressure; and any such ratio as $\frac{\text{B C}}{\text{A C}}$ represents the wetness of the steam. The student will find, if he make various calculations that if the steam is fairly dry to begin with, it becomes wetter on adiabatic expansion, and if it is fairly wet to begin with, it becomes drier on adiabatic expansion.

EX. 9.—The entropy of 1 lb. of water for the absolute Centigrade temperature $t°$ is—

$$\phi = \log_e \frac{t°}{273};$$

Calculate this for two values of the temperature, say, 70° C.

and 170° C. It is, of course, 0 at 0° C. Plot the θ-φ diagram for water. State exactly how much heat is represented by the area of 1 square inch of your diagram.

We have, $$\varphi_w = \log_e \frac{t^\circ}{273},$$

where φ_w means the entropy of the water, and t° absolute Centigrade temperature of the water.

Case I.—70° C. ordinary = 70 + 273 = 343° C. absolute.

$$\therefore \quad \varphi_w = \log_e \tfrac{343}{273} = \log_e 1{\cdot}2564$$
$$= {\cdot}227 \text{ ranks. } \textit{Ans.}$$

Case II.—170° C. ordinary = 170 + 273 = 443° C. absolute.

$$\therefore \quad \varphi_w = \log_e \tfrac{443}{273} = \log_e 1{\cdot}622.$$
$$= {\cdot}483 \text{ ranks. } \textit{Ans.}$$

That is, the entropies of water at 70° C. and 170° C. are ·227 and ·483 ranks respectively. Choose suitable scales for temperature and entropy; say 1 inch to 50° C. and 1 inch to 0·2 rank.

Tabulate the two points found. We have then three points, as follows:—

Temperature. Ordinary C.	Entropy Ranks.
0	0
70	·227
170	·483

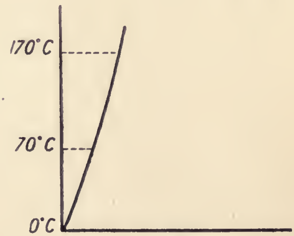

Fig. 10.

Draw a fair curve (which will be nearly a straight line) through the three points (fig. 10).

One square inch on the diagram will therefore be

$$1 \text{ inch} \times 1 \text{ inch} = 50 \times 0{\cdot}2 = 10 \text{ C.H.U.} \quad \textit{Ans.}$$

CHAPTER IV.

SUPERHEATED STEAM.

To the right of the B. of E. diagram will be observed the superheated steam portion of the θ-φ chart. We may now consider how this portion of the diagram is obtained.

The entropy of superheated steam is—

$$\varphi = 0{\cdot}48\,(\log_e \tau_1 - \log_e \tau),$$

where φ means the entropy added to make the steam superheated, τ_1 the absolute temperature after superheating, and τ the absolute temperature of the dry saturated steam.

The formula, therefore, which is of the same general form as the entropy formula for water, gives the excess of entropy possessed by the steam after it is superheated, above what it has when it is dry and saturated.

0·48 is the specific heat of steam at constant pressure.

Example.—Suppose we have 1 lb. of dry saturated steam at 150° C., and that we are going to heat it to 200° C.

Mark point A (fig. 11) on the θ-ϕ diagram corresponding to 1 lb. of dry saturated steam at 150° C. Now take the formula,

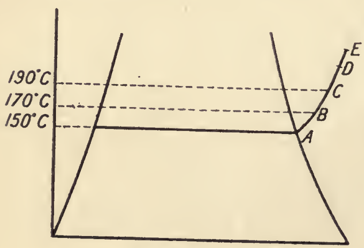

Fig. 11.

$$\varphi = {\cdot}48\,(\log_e \tau_1 - \log_e \tau).$$

$$\tau = 150° \text{ C. ordinary}$$
$$= 150 + 273$$
$$= 423° \text{ C. absolute.}$$

Let us superheat the steam in stages; first of all, say, to 170° C. ordinary = 443° C. absolute. Then τ_1 in the formula is 443.

We have,
$$\varphi = 0{\cdot}48 (\log_e 443 - \log_e 423)$$
$$= 0{\cdot}48 \times {\cdot}04623$$
$$= {\cdot}0222 \text{ ranks.}$$

This is the entropy added to the steam above what it had at 150° C., to superheat it to 170° C. From the chart, the amount of entropy the 1 lb. of dry saturated steam at 150° C. possesses is 1·63 ranks. Add to this the amount of entropy just found—namely, ·0222 ranks—then the total is 1·6522 ranks. Plot this against temperature 170° C., and the point B (fig. 11) on the superheated steam line is obtained.

Now superheat the steam a further 20° C.—that is, to 190° C., or to 463° C. absolute. Then, the entropy the superheated steam now possesses above what it had when dry and saturated at 150° C. is—

$$\varphi = 0{\cdot}48 (\log_e 463 - \log_e 423)$$
$$= 0{\cdot}48 \times {\cdot}09039$$
$$= {\cdot}0434 \text{ ranks.}$$

Add this amount of entropy to the entropy the steam possesses when dry and saturated at 150° C.—namely, 1·63 ranks—and the result is 1·6734 ranks. Plot this against temperature 190° C., and the point C on the superheated steam line is obtained.

Other points can be similarly found on the superheated steam line, and a fair curve drawn through them. It is A B C D E, as shown in fig. 11.

Other superheated steam lines are drawn in a similar manner. If we have 1 lb. of dry saturated steam at, say, 100° C., then τ in the formula $\varphi = 0{\cdot}48 (\log_e \tau_1 - \log_e \tau)$ would refer to the absolute temperature corresponding to 100° C. ordinary.

By means of the process shown, therefore, all the required superheated steam lines may be drawn on the chart.

Problems on Superheated Steam.

EX. 10.—One pound of steam is dry and saturated at 100° C. It is then superheated to 150° C., and then expanded adiabatically to 70° C. Find its dryness after the expansion.

Mark point A (fig. 12) on the saturated steam line corresponding to 1 lb. of steam dry and saturated at 100° C. Go up the *superheated* steam line to B, opposite 150° C. From B draw a vertical line downwards to meet the horizontal line opposite the 70° C. mark, at C. D E is the horizontal line drawn between the water and steam lines opposite 70° C. Then the dryness after the expansion is

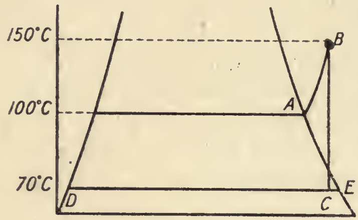

Fig. 12.

$$\frac{\text{D C}}{\text{D E}} = \cdot 97 = 97 \text{ per cent.} \quad \textit{Ans.}$$

EX. 11.—One pound of dry saturated steam at 100° C. is superheated and then expanded adiabatically to 70° C. After the expansion it is again dry and saturated. How many degrees of superheat have been given to it?

Mark point A (fig. 13) on the saturated steam line, corresponding to 1 lb. of steam dry and saturated at 100° C. Mark point B on the saturated steam line corresponding to 1 lb. of steam dry and saturated at 70° C. From B draw a vertical line *upwards*, meeting the superheated steam line from A at C. Read the temperature opposite C. Then this temperature above 100° C. is the number of degrees of superheat given the steam.

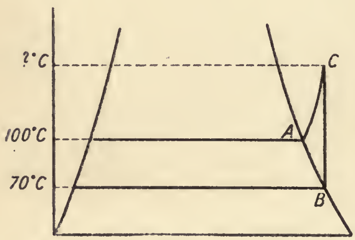

Fig. 13.

Ans. 90° C. of superheat.

EX. 12.—One pound of dry saturated steam at 100° C. is superheated to 200° C. It is then expanded adiabatically to 150° C. Find the number of degrees of superheat it has after the expansion.

Mark point A (fig. 14) corresponding to 1 lb. of dry saturated steam at 100° C. Go up the superheated steam line to B, opposite to 200° C. Draw a vertical line downwards to C, opposite 150° C. Through C draw a superheated steam line C D (if there is such a line passing through C, utilise it; if not, draw a superheated steam line as nearly parallel as possible to the superheat lines on either side of the point). D is the point on the saturated steam line where the superheated steam line from C meets it. Take the temperature corresponding to D. Then the amount of the superheat after the expansion is 150° C. minus this temperature.

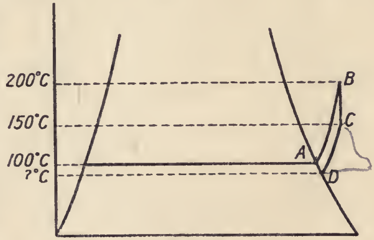

Fig. 14.

Ans. 24° C.

EX. 13.—One pound of steam is dry and saturated at 100° C. It is superheated to 150° C. Find the additional entropy given to it.

Mark point A (fig. 15) on the saturated steam line opposite 100° C. Find the entropy O M corresponding to this point. Go up the superheated steam line to B opposite 150° C. Read the entropy O N corresponding to this point. Then the additional entropy given to the 1 lb. of steam because of the superheating is

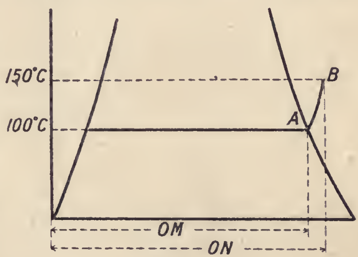

Fig. 15.

$$O N - O M = 1{\cdot}8 - 1{\cdot}74 = {\cdot}06 \text{ ranks.} \quad \textit{Ans.}$$

EX. 14.—One pound of steam is dry and saturated at 125° C. This steam has to be superheated and then expanded adiabatically to 80° C.; after the expansion the steam must be at least 95 per cent. dry. Find the amount of superheat it will be necessary to give it.

Mark point A (fig. 16) on the saturated steam line opposite 125° C. Opposite 80° C. draw a horizontal line C D between the water and steam lines. In this line take a point B such that C B : C D = ·95 : 1. From B draw a vertical line upwards to meet the superheated steam line from A, at E. Read the temperature corresponding to E. Then the amount of superheat necessary is this temperature minus 125° C.

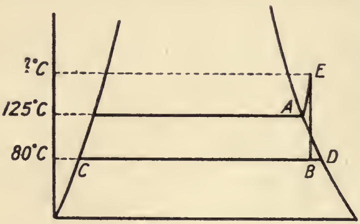

Fig. 16.

Ans. 55° C. of superheat.

EX. 15.—One pound of steam is dry and saturated at 100° C. It is to be superheated until its total entropy is equal to that of 1 lb. of dry saturated steam at 80° C. Find the number of degrees of superheat necessary.

Mark point A (fig. 17) on the saturated steam line opposite 100° C. Mark point B on the saturated steam line opposite 80° C. Read the entropy of the latter. It is 1·81 ranks. From B draw a vertical line upwards to meet the superheated steam line from A at C. Then at C the 1 lb. of superheated steam will have the same entropy—namely, 1·81 ranks—as the 1 lb. of dry saturated steam at B. Read the temperature corresponding to C. Then the number of degrees of superheat required is this temperature minus 100° C. *Ans.* 50° C.

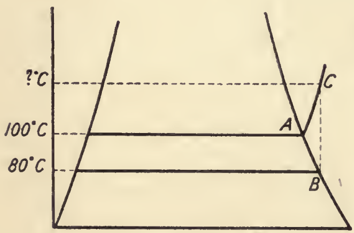

Fig. 17.

EX. 16.—A lb. of water at a temperature τ_1 is converted into dry saturated steam at that temperature; the steam is then superheated to τ_2; it is then expanded adiabatically to τ_3; it is then condensed at τ_3 and afterwards heated to τ_1, thus completing the cycle. Show how to calculate on the θ-φ diagram (1) the work done by the pound of water stuff during the cycle, (2) the efficiency, (3) the work gained through superheating, and (4) the increase of efficiency through superheating.

In fig. 18, mark point A on the water line opposite absolute temperature τ_1; draw a horizontal line A B to the steam line; go up the superheated steam line from B to C, which is opposite absolute temperature τ_2: from C draw a vertical line downwards to meet the horizontal line E H drawn between water and steam lines opposite absolute temperature τ_3; then the cycle of work done by the water stuff is the irregular area E A B C D E.

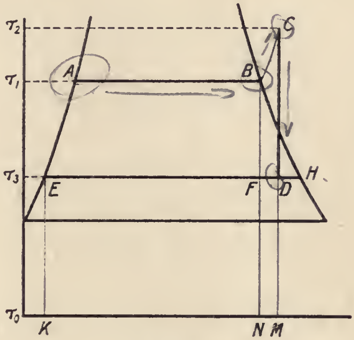

Fig. 18.

We have—

(1) Work done on cycle (in heat units) = area E A B C D E.
Heat units supplied = K E A B C M K.

(2) Therefore efficiency $= \frac{\text{E A B C D E}}{\text{K E A B C M K}}$.

(3) The work gained through superheating is the area B C D F.

(4) If the steam were not superheated, but were just saturated, the work done would be area E A B F E, and the work put in (in heat units), the area K E A B N K. Therefore the efficiency would be $\frac{\text{E A B F E}}{\text{K E A B N K}}$. The student should actually make all these calculations, finding the areas approximately. He will find that superheating increases the efficiency but little. Superheating bestows its benefits indirectly.

CHAPTER V.

Constant Volume Curves on the Entropy-Temperature Chart.

This is the most difficult part of the chart to understand, and great attention should, therefore, be given to it.

Constant Volume.—One pound weight of dry saturated steam occupies a different volume for each different pressure. This can be understood easily, for it is only common sense to suppose that if the steam is under very high pressure, 1 lb. weight of it will occupy a much smaller volume than would be the case if the steam were under only slight pressure. For instance, 1 lb. of dry saturated steam under a pressure of 90 lbs. per square inch occupies a volume of just about 5 cubic feet, whereas 1 lb. of dry saturated steam under a pressure of 220 lbs. per square inch occupies a volume of about 2 cubic feet.

Specific volume means the volume in cubic feet occupied by 1 lb. of dry saturated steam at any assigned pressure.

Suppose we have 1 lb. of dry saturated steam at 90 lbs. per square inch pressure. It occupies just about 5 cubic feet. Now, let the volume remain constant at 5 cubic feet, but let the pressure drop. But how can the pressure drop as long as the volume remains constant ? In only one way. Some of the steam must condense, and be no longer steam, but, in fact, water. If more steam condenses, the pressure is further lowered ; but note, the volume remains the same all the time. This, then, is what we mean by the pressure of steam varying at constant volume.

We may illustrate this pressure drop at constant volume in another way, which will perhaps make it a little clearer.

Suppose we have a cylinder as shown (fig. 19) with a piston P that works up and down in it without friction. Let the cross-sectional area of the cylinder be 1 square foot. Let us have

1 lb. of water in the cylinder. Let the weight of the piston (including atmospheric pressure on the top of it) be $90 \times 144 = 12,960$ lbs. Heat the water until it becomes dry saturated steam. The piston will rise 5 feet. This is because the *pressure on top of the steam* is 90 lbs. per square inch, or 12,960 lbs. altogether, and at this pressure the volume occupied by 1 lb. of dry saturated steam is 5 cubic feet. Now, after the piston has risen its full height, fix it in position so that it can neither rise nor fall further. When the piston has just finished rising, the pressure gauge (fig. 19) will indicate a pressure equivalent to 90 lbs. per square inch absolute. Now remove the source of heat. In a short time the pressure will begin to drop, as shown on the gauge, because some of the steam condenses; the volume, however, remains constant, because the piston is fixed. As more and more steam condenses, the pressure falls further and further, but the volume of what steam there is left is always the same—namely, 5 cubic feet.

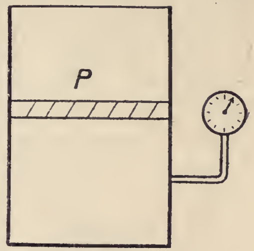

Fig. 19.

This, in fact, is the kind of thing that occurs in the steam engine when " release " takes place. The pressure suddenly drops before the volume has time to appreciably change, and this drop is well shown on the indicator diagram at the part known as the " toe." We shall return to this question of the drop in pressure at constant volume in the cylinder of a steam engine in a later chapter.

In the θ-φ diagram, the constant volume curves represent the loss of entropy due to a drop in pressure at constant volume. It will be seen readily that 1 lb. of dry steam at a pressure of, say, 90 lbs. per square inch represents a certain amount of entropy. If some of the steam condenses while the volume remains constant, there will be a loss of entropy. As more steam condenses at constant volume, there will be a further entropy loss, and so on.

Now consider the construction of the " constant volume " curves on the θ-φ chart.

The most satisfactory way to accomplish this is to divide the horizontal distance, or entropy, between water and steam lines, at any given pressure, into as many equal parts as there are cubic feet of volume in 1 lb. of dry saturated steam at that pressure.

From the steam tables, obtain some particulars like the following :—

Pressure of Steam in lbs. per sq. in.	Vol. of 1 lb. of dry saturated Steam in cubic feet.
30,	13·48
60,	7·00
90,	4·80
120,	3·67

Draw horizontal lines (fig. 20) A B, C D, E F, and G H between

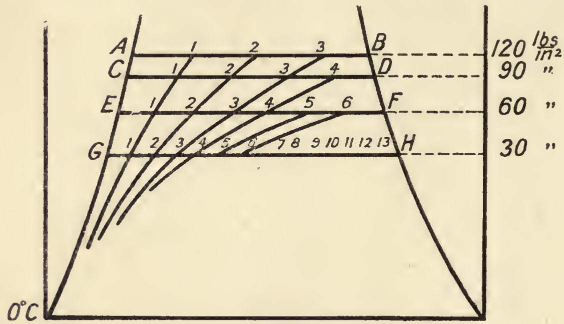

Fig. 20.

the water and steam lines opposite pressures of 120, 90, 60, and 30 lbs. per square inch respectively. Divide the line A B into as many parts as there are cubic feet of steam in 1 lb. of dry saturated steam at a pressure of 120 lbs. per square inch—namely, 3·67. Number the parts, 1, 2, 3, etc. Divide C D into 4·8 equal parts, E F into 7 equal parts, and G H into 13·48 equal parts. In each

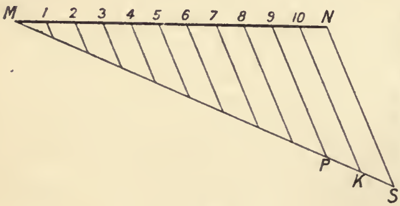

Fig. 21.

case number the parts 1, 2, 3, 4, 5, etc. Now join up by fair curves all the 1's, all the 2's, all the 3's, etc., and we obtain "constant volume" curves.

In the mechanical operation of dividing the lines into equal parts, a geometrical construction should be used. Suppose it is desired to divide the line M N (fig. 21) into a large number, say 11 equal parts. Take a rule and place the zero mark at M. Incline the rule at any acute angle to M N. Mark off 11 lengths of 1 inch or 1 cm., or any other unit (as may be found suitable) by means of the rule. Join the last point so found S, to N. At all the other points, such as K and P, etc., draw lines parallel to S N to meet M N in 10, 9, etc. This method will be found very accurate, and much preferable to dividing by trial.

EX. 17.—One pound of steam is dry and saturated at 150° C. What is its entropy? Some of the steam condenses at constant volume until the pressure drops to 10 lbs. per square inch. Find its loss of entropy.

Mark point A (fig. 22) on the saturated steam line opposite 150° C. Come down the *constant volume line* from A to B, the latter being opposite 10 lbs. per square inch pressure. If there is not a constant volume line at A, one must be drawn as nearly as possible parallel to the curves on either side. It will also be noticed, that for convenience in tracing these curves, the line which goes down the centre of the chart is subdivided between each pair of curves. Read the entropy at A and at B, and the difference is, of course, the loss. *Ans.* 1·08 ranks.

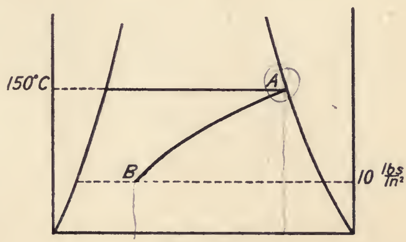

Fig. 22.

EX. 18.—One pound of steam is dry and saturated at 200° C. What is its specific volume? Some of the steam condenses at constant volume, until it loses half its entropy. It then expands adiabatically to 60° C. Find its dryness after the expansion.

Mark point A (fig. 23) on the saturated steam line opposite 200° C. Come down a constant volume curve to the point B (at B the steam must have only half the entropy it has at A). From B draw a vertical line downwards to E. E is on the horizontal line C D, drawn opposite 60° C. between the water and steam lines. Then the dryness after the expansion is C E divided by C D. *Ans.* 33·7 per cent.

Observe very carefully that the constant volume curves are only to be used when there is actual condensation going on at *constant volume.* There are many questions involving volumes that can be solved by means of the chart, which have nothing whatever to do with the constant volume curves. The following examples, taken from B. of E. papers, will illustrate this.

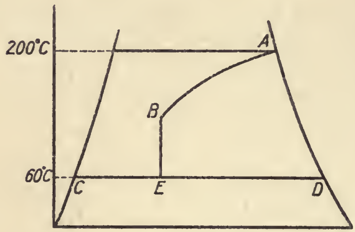

Fig. 23.

EX. 19.—One pound of stuff consisting of 0·6 lb. steam and 0·4 lb. water expands adiabatically from 311° F. to 230° F. What is the weight of water at the end? What are the volumes v of steam at the beginning and at the end? Neglect volumes of water.

Mark point A (fig. 24) on the horizontal line B C, opposite 311° F., such that $\frac{BA}{BC} = \cdot 6$. From A draw a vertical line downwards to D. D is on the line E F drawn horizontally at 230° F. The weight of water at the end of the expansion is then $\frac{DF}{EF}$ of 1 lb.

Ans. 0·42 lbs.

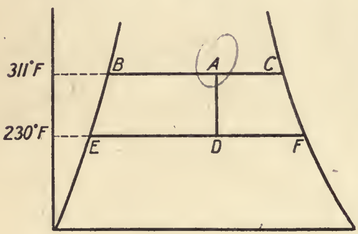

Fig. 24.

One pound of dry saturated steam at 311° F. occupies 5·433 cubic feet; but, as there is only ·6 lb. of steam present, it will occupy 5·433 × ·6 = 3·2598 cubic feet. *Ans.*

One pound of dry saturated steam at 230° F. occupies 19·03 cubic feet; but because, after the expansion, there is only ·58 lb. of steam present, it will occupy 19·03 × ·58 = 11·037 cubic feet. *Ans.*

EX. 20.—Find the entropy of 1 lb. of wet steam. 20 per cent.

of which is water at 160° C. Dry steam at this temperature measures 4·827 cubic feet to the pound. If at 100° C. (p = 14·7 lbs. per square inch; u = 26·43 cubic feet per pound) it has the same entropy, how much of it is water? What are the volumes in these two cases, neglecting the volume of water?

Draw B C (fig. 25) opposite 160° C. Mark point A such that B A : B C = ·8 : 1. Draw E F opposite 100° C. The steam after expansion has to have the same entropy as before, hence the expansion must be adiabatic. Therefore, from A draw the adiabatic line A D. The amount of water after the expansion is $\frac{DF}{EF}$ of 1 lb.

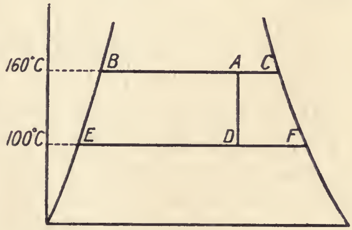

Fig. 25.

Ans. ·26 lb. water.

Volume of 1 lb. of dry saturated steam at 160° C. = 4·827 cubic feet; but as there is only ·8 lb. steam, the volume is 4·827 × ·8 = 3·816 cubic feet. *Ans.*

Volume of 1 lb. of dry saturated steam at 100° C. = 26·43 cubic feet; but as there is only ·74 lb. of steam, the volume is 26·43 × ·74 = 19·558 cubic feet. *Ans.*

CHAPTER VI.

THE "PERFECT" HEAT ENGINE REPRESENTED ON THE ENTROPY-TEMPERATURE CHART.

The Carnot Cycle.—Carnot's engine is the most efficient imaginable. It is perfectly reversible.

The cycle of operations is as follows, and will be considered first with reference to the ordinary pressure-volume chart. Of course, the student is assumed to be already familiar with this part of the work, and all that can be said here is a few words to bring before his memory the main operations.

At A (fig. 26) we have 1 lb. of water at a temperature of τ_1 absolute. We first evaporate it into dry saturated steam, with the volume A B. This is an isothermal operation. Next, we expand it adiabatically to C, until its temperature falls to τ_2 absolute. It is now compressed at this temperature to a volume O D (this is an isothermal operation), the point D being such, that when from D it is compressed adiabatically to τ_1, it occupies its original volume, and the cycle of operations is complete. This cycle, then, consists of two isothermal and two adiabatic operations.

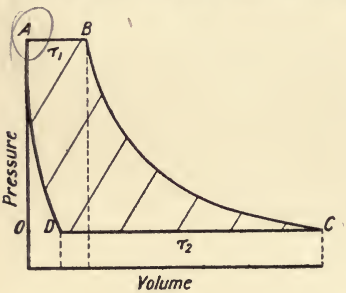

Fig. 26.

The efficiency of the cycle is $\frac{\tau_1 - \tau_2}{\tau_1}$.

The work done per cycle equals the sectioned area.

Now, in the *p-v* diagram, although the work done in the Carnot cycle is shown, neither the heat received or rejected, nor the efficiency is indicated. Consequently, to obtain these quantities mathematically would be a tedious process. On the θ-φ chart, however, they can be obtained at once.

To find the efficiency of an ideal engine working on the Carnot cycle, by means of the θ-φ diagram—

The point A on the θ-φ chart (fig. 27) represents 1 lb. of water at an absolute temperature τ_1. Its entropy is O E ranks. First operation; it is evaporated into 1 lb. of dry steam at temperature τ_1, when its entropy becomes O F; this is the first isothermal process. Second operation: it is expanded adiabatically at B to C, until the temperature falls to τ_2 absolute; the amount of heat it contains is, however, neither increased nor diminished; its entropy is the same at C as at B; this is the first adiabatic process. Third operation: at C it is compressed isothermally at temperature τ_2 absolute, to D; it gives up much heat, and accordingly its entropy falls from O F to O E; this is the second isothermal process. Fourth operation: at D it is compressed adiabatically to A, the entropy remaining constant, but the temperature rising from τ_2 to τ_1; this is the second adiabatic process, and completes the cycle.

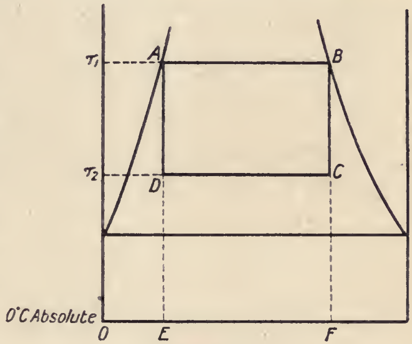

Fig. 27.

The work done during the cycle is given by the area of the rectangle A B C D (of course, in heat units).

The work received (in heat units) is given by the area of the rectangle A B F E (where the bottom line of the chart represents the zero of absolute temperature, and, of course, if drawn to scale, should be much lower down than shown).

The work rejected (in heat units) is given by the area of the rectangle D C F E.

$$\text{The efficiency} = \frac{\text{work done}}{\text{work received}} = \frac{\text{A B C D}}{\text{A B F E}} = \frac{\tau_1 - \tau_2}{\tau_1}.$$

By the use, therefore, of the θ-φ chart, it is much easier to find the efficiency of the Carnot cycle than by any other method.

The Rankine Cycle.—Many engineers prefer to use this as an ideal cycle with which to compare the working of ordinary steam engines.

This cycle differs from the Carnot, in that the 1 lb. of water is *not*, at the commencement of the series of operations, at the temperature at which it is converted into steam. The water is at some lower temperature, and requires to be heated to the boiling temperature. In fig. 28, A represents 1 lb. of water at a temperature τ_2 absolute. It is heated to τ_1 absolute (B), where it is converted into dry saturated steam (C); it is then expanded adiabatically to τ_2 absolute (D), and then

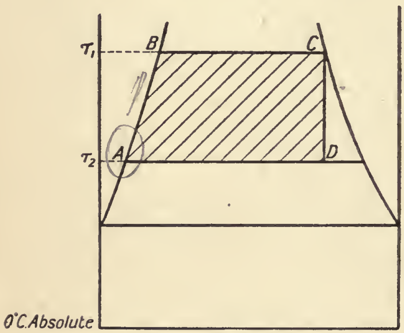

Fig. 28

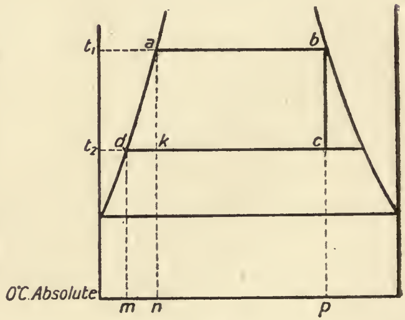

Fig. 29.

compressed isothermally at τ_2 to A, which completes the cycle of operations—that is, it becomes water again at A. The sectioned area of the figure gives the work done on the Rankine cycle, and it will be noticed that this work is greater than with the Carnot cycle.

EX. 21.—Rankine cycle, perfect steam engine, with dry steam at t_1, expanded adiabatically to t_2; find a formula for the work done per pound of steam. How do we find the answer graphically? (B. of E., Hons., Part 2, 1899.)

In fig. 29, steam begins to be formed at *a*, and *ab* is the change of entropy because of this; the adiabatic expansion is represented by *bc*, and *cd* is the process of condensation which completes the cycle.

Heat taken in during the warming of the feed water is area *m d a n*.

Heat taken in during evaporation is the area *n a b p*.

The work done is the enclosed area *d a b c d*.

The heat rejected is the area *p c d m*.

The latter part of the question (that to do with entropy) asks how the work done is obtained graphically. It would be probably sufficient to say, by taking the area *a b c d a* by calculation or by the planimeter. However, to analyse the question a little further, the formula for the work done on this cycle is as follows :—

$$\text{Work done} = \Sigma \frac{\delta\theta(\tau - \tau_2)}{\tau} + \frac{L_1}{\tau_1}(\tau_1 - \tau_2),$$

where Σ means "the sum of such terms as."

$\delta\theta$ means a very small quantity of heat taken in.

τ means any absolute temperature between τ_1 and τ_2.

τ_1 means the highest absolute temperature.

τ_2 means the lowest absolute temperature.

L_1 means the latent heat at τ_1.

Look at the diagram. The area *a b c d a* consists of the areas of rectangle *a b c k* and figure *a k d* (very nearly a triangle).

The first portion of the above formula—namely, $\Sigma \frac{\delta\theta(\tau - \tau_2)}{\tau}$ —is the area of the figure *a k d*. Presuming a knowledge of integration, it will easily be seen how this is so. Otherwise, this portion of the formula is better left alone for the present.

The second portion of the formula—namely, $\frac{L_1}{\tau_1}(\tau_1 - \tau_2)$ is the area of the rectangle *a b c k*. This is easily seen, because *ab* is $\frac{L_1}{\tau_1}$ (from definition), and *ak* is $\tau_1 - \tau_2$.

EX. 22.—A perfect steam engine, Rankine cycle; given the higher and lower temperatures and initial wetness or amount of superheating. Using a θ-φ diagram, show how you would find the work done per pound of stuff. If the stuff is released before the end of the expansion, show the amount of lessening of work done. (B. of E., Hons., Part 1, 1899.)

This question involves several important applications of the chart, and should be studied carefully.

Case I.—Let t_1 and t_2 (fig. 30) be the higher and lower absolute temperatures respectively. Mark point *a* on the water line opposite t_2. Go up the water line to *b*, which is opposite t_1. Draw *bc* between water and steam lines, and divide *bc* at *e*, such that *be* : *bc* represents the required dryness. Draw the adiabatic line *ef*, *f* being in *ak* drawn between water and steam lines. The complete Rankine cycle for

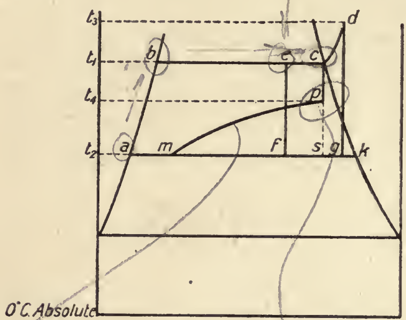

Fig. 30.

the wet steam is then *a b e f a*, and the work done is the area of this figure, which may be found by planimeter, dividing up into a rectangle and approximate triangle, or by the thermodynamical formula.

Case II.—Given the amount of superheating. Mark point *a* (fig. 30) on the water line opposite t_2. Go up the water line to *b*, opposite t_1. Draw *bc* horizontally between the water and steam lines. From *c* go up the superheated steam line to *d*, which is opposite t_3, the absolute temperature to which the superheating is to be carried. From *d* draw vertically downwards the adiabatic line *dg*. *g* is a point in *ak* drawn horizontally between the water and steam lines. The complete Rankine cycle is then the figure *a b c d g a*, and the area of this is the work done per pound of stuff.

Case III.—Stuff released before end of expansion.

Mark point *a* on the water line (fig. 30) opposite t_2. Go up the water line to *b* opposite t_1. Draw horizontal line *bc* between water and steam lines. Suppose now that the steam is dry before expansion begins. We must, therefore, commence the adiabatic expansion from *c*, on the steam line. From *c* draw a vertical line *cp* downwards. *p* is opposite t_4, to which temperature it will be supposed the expansion is carried. Release now occurs (before the adiabatic expansion is complete), so from *p* we must proceed along the *constant volume curve* to *m*, which is opposite t_2. The reason for this is, that when release takes place, the steam drops in pressure at *constant volume*. From *m* draw a horizontal line to *a*. The cycle is then the figure *a b c p m a*, and the area of this gives the work done per pound.

If release did *not* occur before the end of expansion, the work done per pound of stuff would be the area of the figure *a b c s a* (the Rankine cycle). So that the lessening of work done because of incomplete expansion is the area of the figure *p m s*.

EX. 23.—Find the work that would be done by 1 lb. of steam 90 per cent. dry at 160° C. on the Rankine cycle, the lower temperature being 100° C. Your answer must be correct to 3 per cent.

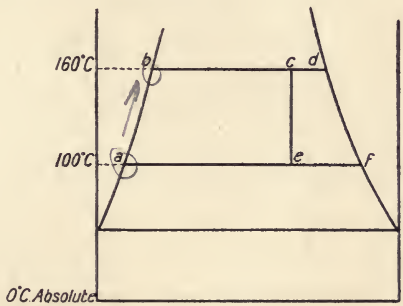

Fig. 31

Mark point *a* (fig. 31) on the water line opposite 100° C. Go up the water line to *b*, opposite 160° C. Draw *bd* horizontally between water and steam lines. Take *c* in *bd*, such that *bc* is $\frac{9}{10}$ of *bd*. Draw a vertical line downwards to meet *a f*, drawn horizontally between water and steam lines, in *e*. Then the Rankine cycle is the figure *a b c e a*, and its area should be calculated by planimeter or by approximate means. This area is the work done (in C.H.U.) per pound of stuff.

EX. 24.—A pound of water-steam at 160° C. is supplied to a turbine, and, exhausting at 40° C., is found to be of dryness 0·95; the work done per pound of steam is less than what the Rankine cycle gives; how much less? Assume no radiation of heat from the turbine.

A pound of steam at 160° C. and 0·9 dry is shown at C (fig. 32). It expands adiabatically to D where it is about 0·73 dry. The Rankine cycle is the area A B C D A, and the area of this figure is the work done by the perfect steam engine.

Now, in the actual turbine, the expansion is not adiabatic because of give and take of heat between the steam and the casing of the turbine. Nevertheless, as there is no loss by radiation, all the heat has to be accounted for. In this case the steam is found at 40° C. to be ·95 dry. This is shown at G. The whole work done by the turbine is, therefore, the Rankine area A B C D A minus the area of the rectangle D G H F

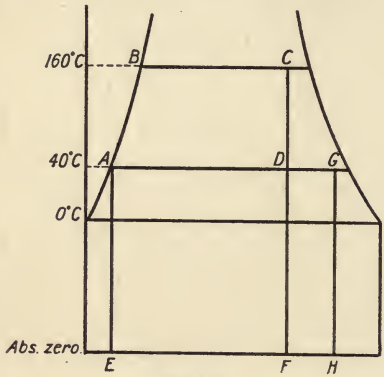

Fig. 32.

(down to absolute zero). The reasoning is as follows:—The total heat given to a pound of water at 40° C. to convert it into steam at 160° C. and 0·9 dry, is the area E A B C D F E (to absolute zero), and steam at 40° C. and dryness 0·73 carries away heat equal to area E A D F E. Steam at 40° C. and dryness 0·95 carries away heat E A G H E. The turbine, therefore, carries away more heat than the Rankine engine equivalent to area F D G H. As they both receive the same total heat, the turbine, therefore, does less work than the Rankine engine by an amount equal to the area F D G H.

CHAPTER VII.

The Indicator Diagram compared with the Entropy-Temperature Diagram.

Note.—The method of treating this portion of the subject has been, with permission, largely based upon that of Professor Ewing in his book, _The Steam Engine and Other Heat Engines._

To convert the *expansion* part of an indicator diagram into a corresponding θ-φ diagram.

Before this can be done, the dryness of the steam on the expansion stroke must be known. As, in the ordinary indicator diagram, the expansion curve is not for dry steam, but for steam and water, the dryness is obtained as follows :—Imagine that the piston has moved some little distance along its stroke, and that expansion has begun. The working substance behind the piston is then a mixture of steam and water. The amount of this is the steam given to the cylinder at the beginning of the stroke (the cylinder feed) from the boiler, plus the steam that has been left in the cylinder at the end of the last stroke (the cushion steam). It becomes necessary, therefore, to find the weight of this cushion steam. To accomplish this, examine the indicator diagram, and take on the diagram a point D, after compression has commenced (fig. 33), and after the exhaust valve has become completely closed. Note the pressure and volume at this point. The true volume is the uncompleted part of the stroke plus the clearance volume. Now, if it is assumed that the cushion steam at D is dry, which it probably is, a certain volume of dry steam at a certain pressure has been found. But at any particular pressure 1 lb. of steam occupies a particular volume, hence it is an easy matter to deduce the weight of the cushion steam.

The cylinder feed is the weight of steam that enters the cylinder each stroke from the boiler. This can be obtained from a measurement of the water that enters the boiler over a considerable period of time, or from a similar measurement of the condensed steam.

We have now, at the required point on the expansion stroke, the total weight of steam and water behind the piston—namely, cylinder feed and cushion steam. Next proceed to draw on the indicator diagram a *Saturation* Curve. This shows the volume occupied by the *total quantity* of water and steam if the whole were dry saturated steam, at each pressure reached during expansion. We can then easily obtain the dryness of the steam at any point during the expansion.

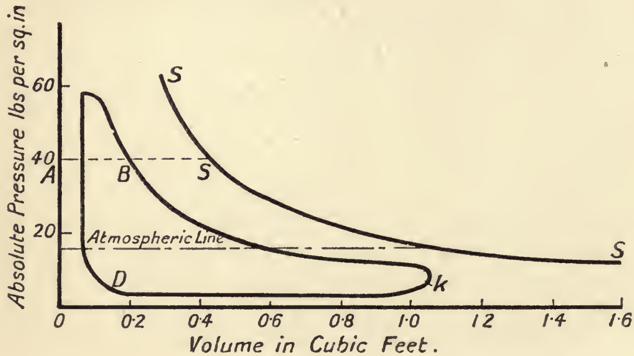

Fig. 33.

Before, then, the expansion curve of the indicator diagram can be converted into a θ-φ diagram, we require to know :—

(1) The size of the cylinder (in order to obtain the volume of water-stuff given to the cylinder each stroke—*i.e.*, the cylinder feed).

(2) The clearance volume (in order to obtain the exact weight of steam expanding behind the piston).

(3) A *mean* indicator diagram (in order to get an *average* value of the expansion curve, etc.).

From these data a saturation curve for the expansion is drawn on the indicator diagram—*i.e.*, a curve showing at each point the exact volume occupied by the weight of steam expanding, if it were dry and saturated.

As an example, consider the indicator diagram shown (fig. 33) taken from an actual engine test by Professor Ewing. In this diagram the axis of no volume is to the left, by a distance which represents the volume of the clearance in proportion as the whole length of the diagram represents the volume of the piston stroke. This volume has to be found either by calculation, or by direct measurement by filling the ports, etc., with water. The saturation curve S S is then drawn as explained above. Then, if a horizontal line A B S be drawn to intersect the expansion curve at any point B, A B is the actual volume which the expanding mixture filled at this pressure, A S is the volume it would have filled if dry and saturated, and B S is the volume that is lost by wetness. Hence the dryness is $\frac{AB}{AS}$. In this way the dryness is found at any stage of the expansion. In this practical test the amount of cylinder feed per single stroke was 0·0404 lb. (This was obtained by measuring the feed water over a considerable period of time.) The pressure at the point D was found to be 4 lbs. per square inch, and the volume there was 0·12 cubic feet. Since the volume of 1 lb. at that pressure is 90·4 cubic feet, it follows that the amount of cushion steam was $\frac{\cdot 12}{90\cdot 4} = 0\cdot 0013$ lb. This gives a total of $\cdot 0404 + \cdot 0013 = \cdot 0417$ lb., for which the curve S S is drawn. By measuring the values of $\frac{BS}{AS}$ at points along the curve, it is found that the proportion of water in the mixture was 52 per cent. at cut-off, then increased to about 55 per cent., and finally sank to 37 per cent. just before release.

To convert the expansion part of the p-v diagram into the θ-ϕ diagram.

In the θ-ϕ diagram (fig. 34) let $a\,b$ be drawn at the temperature which corresponds to the pressure at the point of cut-off, and let it be divided at c, so that $\frac{ac}{ab}$ represents the dryness of the steam as found from the indicator diagram. Similarly at any lower temperatures reached during expansion, let lines $a'\,b'$, $a''\,b''$, be divided at points c', c'', in the proportion of the dryness of the steam at these temperatures, as found from the indicator diagram. In this way the curve $c\,c'\,c''$ is obtained, which represents the real process of expansion, and this is readily compared with the ideal adiabatic process represented by the straight vertical line $c\,g$. Taking c'' as the point of release, the diagram may be continued by drawing a constant-

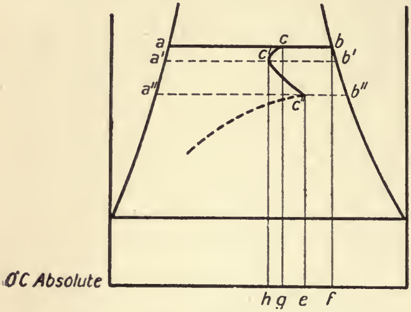

Fig. 34.

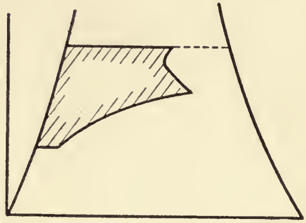

Fig. 35.

volume curve from c'' down to the temperature of the exhaust (as explained in the previous chapter). The completed θ-φ diagram would then be as shown in fig. 35.

Some interesting results can be obtained from fig. 34. In the

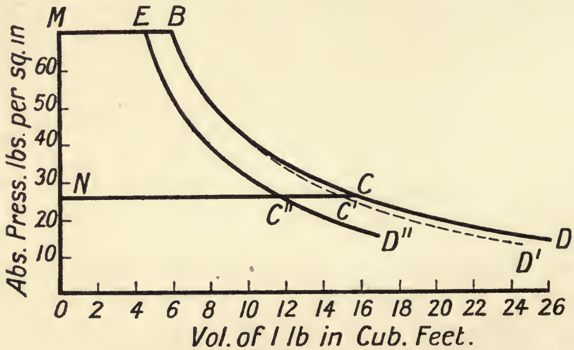

Fig. 36.

particular expansion shown here we see a very customary action going on in the cylinder. During the expansion from c to c', the proportion of water in the cylinder is increasing (because the wet steam ratio is becoming greater and greater). This means that the cylinder walls must be taking away heat from the steam, the amount being $c\,g\,h\,c'$. At point c' the curve changes its direction and the steam becomes continually drier, which means that the steam is now taking heat from the cylinder walls, the whole amount recovered up to the point of release being the area $c'\,c''\,e\,h$.

To draw on the indicator diagram the curve for adiabatic expansion. From the θ-φ diagram, and from the knowledge of the relation of pressure to volume in saturated steam (which can be obtained from the tables) it is easy to determine what proportion of water will be present at any stage in adiabatic expansion or compression.

Thus let B C D (fig. 36) be a portion of the pressure-volume curve for saturated steam. To draw the adiabatic curve from any assigned point B, refer to the tables to find the temperature which corresponds to the assigned pressure at B, and draw a horizontal line $o\,b$ at that temperature on the θ-φ chart (fig. 37). If the steam is assumed to be dry at b, draw a vertical line $b\,c'$ through b. Taking any lower pressure, draw the horizontal line N C in the pressure-volume diagram (fig. 36), refer to the tables for corresponding temperature, and draw the line for that temperature, $p\,c$, on the θ-φ chart. Measure the ratio $\frac{p\,c'}{p\,c}$. This is the dryness. Take a point C′ in N C (fig. 36) such that $\frac{\text{N C}'}{\text{N C}} = \frac{p\,c'}{p\,c}$. Then C′ is a point on the adiabatic curve. The same construction is to be repeated to find as many points as are sufficient to let the curve be drawn. If the steam is wet to begin with, the initial volume of 1 lb. will have some value M E less than M B, and the curve of adiabatic expansion starts from E. It is found in that case by taking e in fig. 37, so that $\frac{o\,e}{o\,b} = \frac{\text{M E}}{\text{M B}}$ (the initial dryness), drawing the vertical $e\,c''$, and taking C″ in the figure, so that $\frac{\text{N C}''}{\text{N C}} = \frac{p\,c''}{p\,c}$,

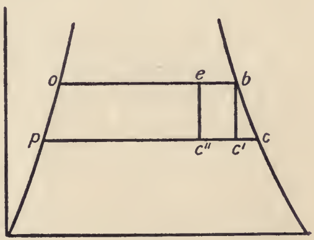

Fig. 37.

this ratio being the dryness after the adiabatic expansion has brought the pressure of the mixture down to the level of pressure N C; C″ is then a point on the required curve. The curve E C″ D″ has been sketched in this way to show the adiabatic expansion of steam containing 25 per cent. of moisture to begin with.

EX. 25.—Sketch an indicator diagram such as might be expected from a non-condensing engine with a common slide valve. How do we show on it the dryness of the steam during expansion? What information must be given to enable us to do this? Show how to convert the expansion part into a θ-φ diagram. (B. of E., Hons., Part 1, 1899.)

To show the dryness of the steam during expansion, we must draw a saturation curve.

The information we require for this is:—

(1) The cylinder feed, or the weight of steam that enters the cylinder per stroke. This may be obtained by weighing the feed water over a long period.

(2) The weight of cushion steam. To get this, we take a point on the diagram such as D (fig. 33) after compression has begun, and read off the pressure and volume there. We look up in the tables the volume of 1 lb. of dry steam at the required pressure. Suppose it were 80 cubic feet per lb.; and that the volume at the same pressure, from the diagram, is 5 cubic feet. Then obviously there must be only $\frac{5}{80} = \frac{1}{16}$ lb. of steam present (if we suppose that the cushion steam is dry, which it probably is). We now add the cylinder feed and the cushion steam together, and the sum gives us the weight of steam that expands. Suppose it is dry at cut-off. We now draw the saturation curve. To accomplish this, we simply read from the tables the volume in cubic feet occupied by 1 lb. of the steam at the various pressures during expansion as given on the indicator diagram. Let S S be the saturation curve (fig. 33). Then if we draw any horizontal line such as A B S, the dryness of the steam is $\frac{AB}{AS}$.

To convert the expansion part into the corresponding part on the θ-φ chart, see the method given in the early part of the chapter.

EX. 26.—On an indicator diagram there is a point on the expansion curve where the absolute pressure is 85 lbs. per square inch. If at this point there is a weight of water in the cylinder just equal to the weight of the steam, show on a θ-φ diagram how the stuff changes during expansion. What is the total heat gained or lost by it from $p = 85$ to $p = 25$? You are given the following information :—

$p \frac{\text{Lbs.}}{\text{Ins.}^2}$	θ° C.	φ for 1 Lb. of Water.	φ for 1 Lb. of Steam.	U, in Cubic Feet, the Volume of 1 Lb. of Dry Steam.
85	158	0·461	1·608	5·07
50	138·2	0·415	1·649	8·34
25	115·6	0·356	1·704	15·99

(B. of E., Stage III., 1904.)

The first thing to do is to draw, if possible, the steam saturation curve on the indicator diagram. Through A (fig. 38) opposite

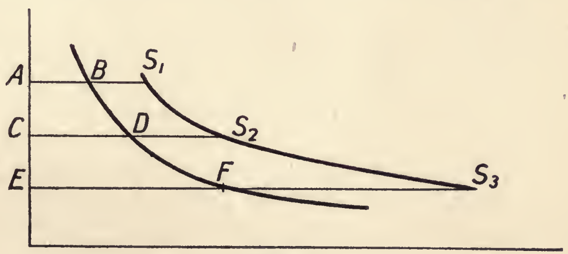

Fig. 38.

85 lbs. per square inch draw a horizontal line A B S_1. Make B S_1 = A B. Then the dryness of the steam at B on the expansion curve = ·5, which we are given. Now, the volume of 1 lb. of dry saturated steam at this pressure is 5·07 cubic feet; make, therefore, A S_1 to represent 5·07 cubic feet to any scale. Opposite a pressure of 50 lbs. per square inch draw the horizontal line C D S_2; the volume of 1 lb. of dry saturated steam at this pressure is 8·34 cubic feet; therefore, C S_2 must be made equal to this (to the same scale to which A S_1 represents 5·07 cubic

feet). Opposite a pressure of 25 lbs. per square inch draw a horizontal line E F S_3. E S_3 must represent 15·99 cubic feet. Join S_1, S_2, S_3 by a fair curve, and we have the saturated steam curve.

To transfer to a θ-φ diagram there is no difficulty. Opposite the three given temperatures corresponding to the three pressures (fig. 39) draw, between the water and steam curves, horizontal lines *a b*, *c d*, and *e f*. Mark a point *g* in *a b* such that $\frac{a\,g}{a\,b}$ gives the dryness. This dryness is obtained from the fraction $\frac{\text{A B}}{\text{A S}_1}$ on the indicator diagram. Find *h* and *j*, on *c d* and *e f*, in the same way. Join up by a fair curve points *g h j*. We then have the expansion curve shown on the θ-φ chart. The total heat given out by the steam during expansion from *g* to *j* is the area of the figure *g h j m n* (down to absolute zero).

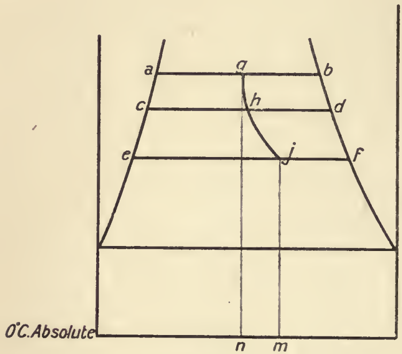

Fig. 39.

A method has been given for transferring the expansion part of an indicator diagram to the θ-φ diagram. For this purpose we had first to draw a saturation curve. Sometimes, however, it becomes necessary to transfer points to the θ-φ diagram that are not on the expansion part of the stroke, and to do this we have to proceed differently.

To transfer points on the indicator diagram not on the expansion curve, to the θ-φ diagram.

First, it is necessary to find the *diagram factor* of the indicator diagram. The diagram factor is the number by which the actual weight of steam in the cylinder must be multiplied in order to make it exactly one pound. For instance, if the compression steam plus the steam expanding in the cylinder exactly equal one pound, the diagram factor is one; but if this total weight were, say, $\frac{3}{4}$ pound, the diagram factor would be $\frac{4}{3}$, because

$$\tfrac{3}{4} \times \tfrac{4}{3} = 1.$$

$$\text{The diagram factor} = \frac{1}{\text{actual weight}}$$

$$= \frac{1}{\frac{3}{4}}$$

$$= \frac{4}{3}.$$

After finding the diagram factor, take any point such as k on the indicator diagram (fig. 33), not on the expansion curve; then find the volume of steam in the cylinder at that point, by direct measurement from the indicator diagram; multiply this volume by the diagram factor, and the product gives the position of the point k as to volume on the θ φ chart.

For the volume at $k \times$ diagram factor gives the constant-volume line on which k will lie on the θ-φ chart; also, we know the horizontal line on which k lies, because we know the pressure at point k from the indicator diagram; hence k on the θ-φ chart lies at the intersection of the constant-volume line found and the horizontal pressure line (fig. 40).

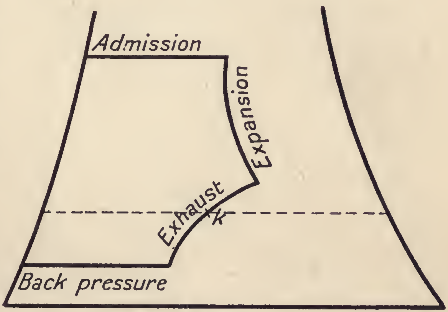

Fig. 40.

Any other point on the indicator diagram may be transferred to the θ-φ diagram in the same way, and, if it is desired, sufficient points may be transferred, to convert the complete indicator diagram into a complete θ-φ diagram.

CHAPTER VIII.

Some Miscellaneous Examples.

EX. 27.—Show how the θ-φ diagram of a steam engine with incomplete expansion differs from that of an ideal engine working on the Carnot cycle. Explain how the θ-φ diagram may be used to show the action of the cylinder walls during expansion. (I.C.E., 1897.)

In fig. 41, the steam expands adiabatically at c to a point d; then instead of expanding further it condenses *at constant volume* (in other words, the exhaust is open), so that $d\,e$ is a line of

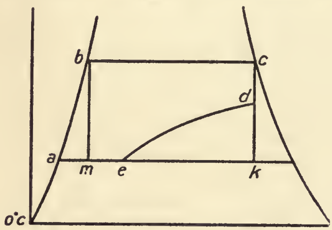

Fig. 41.

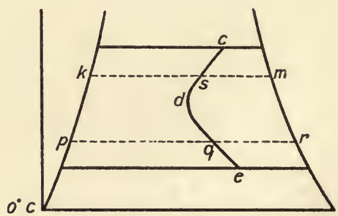

Fig. 42.

constant volume; the next process is $e\,a$, where the condensation is completed at exhaust temperature. The cycle is $a\,b\,c\,d\,e\,a$. The Carnot cycle is $b\;c\;k\;m\;b$.

In fig. 42, let $c\,d\,e$ be the expansion curve on the θ-φ chart. From c to d, the steam is becoming wetter, because any dry steam ratio such as $\frac{k\,s}{k\,m}$ is becoming less; hence up to the point d the cylinder walls are abstracting heat from the steam. From point d onwards the steam is becoming drier, because any dry steam ratio such as $\frac{p\,q}{p\,r}$ is becoming greater; hence during this part of the stroke the steam is abstracting heat from the cylinder walls.

EX. 28.—Show that the heat supplied during the expansion of a mixture of steam and water is graphically represented on a θ-φ diagram. Show that if no heat is supplied to steam which is originally dry, it necessarily condenses during expansion, and exhibit graphically the heat necessary to prevent condensation. (I.C.E., 1898.)

We have,

$$\text{change of entropy} = \frac{\text{small quantity of heat}}{\text{absolute temperature}},$$

hence,

$$\left.\begin{array}{c}\text{small quantity}\\ \text{of heat}\end{array}\right\} = \text{absolute temperature} \times \text{change of entropy}$$

$$= \text{area of a part of diagram.}$$

Hence, generally, the area of the θ-φ diagram represents heat units.

If the steam is dry at c (fig. 43), and no heat is given to

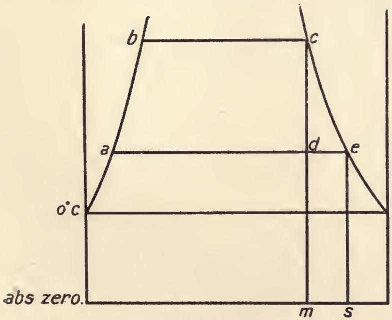

Fig. 43.

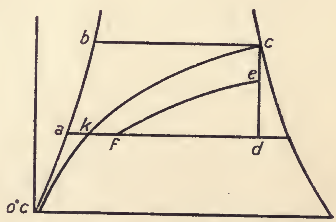

Fig. 44.

it, it may expand like $c\,d$ (adiabatic). If heat were continually given to it so as to keep it dry during expansion, d would fall upon e. The extra heat units, therefore, necessary to keep the steam dry during expansion is given by the area $m\,d\,c\,e\,s$ (down to absolute zero).

EX. 29.—On a θ-φ diagram show clearly the cycle of engines working on (a) the Rankine cycle, (b) the Rankine cycle but with incomplete expansion, (c) non - expansive cycle. (I.C.E., 1903.)

In fig. 44, (*a*) area *a b c d a* is the Rankine cycle.

(*b*) Area *a b c e f a* is the Rankine cycle with incomplete expansion.

(*c*) Area *a b c k a* is the non-expansive cycle—curves *e f* and *c k* are those of constant volume.

EX. 30.—Show how the θ-φ diagram may be used to obtain the following :—(1) The heat units required to raise 1 lb. of water from any temperature $t_1°$, to any higher temperature $t_2°$; (2) the heat units required to convert the 1 lb. of water at $t_2°$ into 1 lb. of dry saturated steam at $t_2°$; (3) the sum of (1) and (2)—*i.e.*, the total heat; (4) the intrinsic energy of the 1 lb. of steam; (5) the external energy of the 1 lb. of steam.

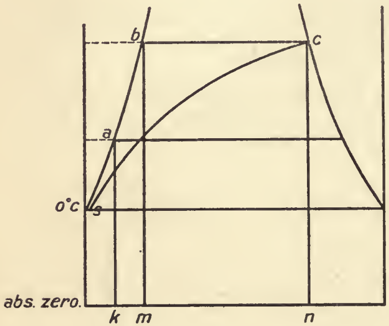

Fig. 45.

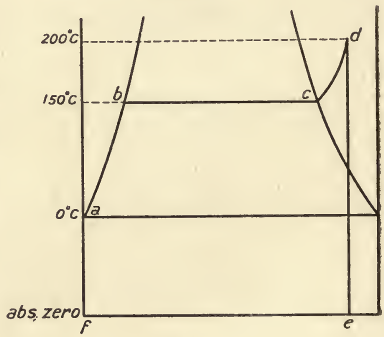

Fig. 46.

In fig. 45, (1) the sensible heat is the area *a b m k a* (down to absolute zero).

(2) The latent heat is the area *b c n m b* (down to absolute zero).

(3) The total heat in the area *a b c n k a* (down to absolute zero).

(4) The intrinsic energy is the area *a b c n k a* minus the area *s a b c s* (where the curve *c s* is the curve of constant volume).

(5) The external energy is the area *s a b c s*.

EX. 31.—Find from the θ-φ chart the total heat required to raise 1 lb. of water from 0° C. to 150° C., to convert it into dry saturated steam at that temperature, and then to superheat it 50° C.

In fig. 46, if *a b c d* represents the series of operations, the total heat required is the area *a b c d e f a* (down to absolute zero).

EX. 32.—Steam 80 per cent. dry at 100 lbs. per square inch escapes adiabatically to a place where the pressure is 20 lbs. per square inch. Find its velocity.

In fig. 47, area *d a k c d* is the work stored up in 1 lb. of the steam = its kinetic energy $= \frac{v^2}{2g}$, hence $v^2 = 2g \times$ area *d a k c d*.

Hence to get v, the velocity, bring the area *d a k c d* to foot-lbs. and multiply by 64·4, and take the square root of the product.

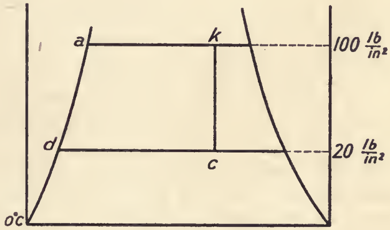

Fig. 47.

EX. 33.—A lb. of superheated steam at rest at t_1° C. flows adiabatically to a place where the temperature is t_2° C.; to find the velocity at the lower temperature.

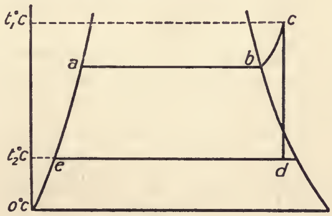

Fig. 48.

In fig. 48, area *a b c d e a* = work stored up in 1 lb. of steam

$$= \text{its kinetic energy} = \frac{v^2}{2g},$$

$$\therefore \frac{v^2}{2g} = \text{area } a\,b\,c\,d\,e\,a \text{ (in foot-lbs.)}$$

$$\therefore v = \sqrt{2g \times \text{area } a\,b\,c\,d\,e\,a}.$$

Hence to get v, find area *a b c d e a*, bring it to foot-lbs. multiply by 64·4 ($2g$), and extract the square root.

APPENDIX.

Questions from Examination Papers, with Helps for their Solutions.

1. To what uses do we put a t, θ diagram? What is entropy? What is the difference between the entropies of a pound of steam and a pound of water at the same temperature? (B. of E., Advanced, 1898.)

Solution.—See Chapter II., and Ex. 3, Chapter III.

2. A pound of water at 0° C. is heated as water to 150° C., and then converted into wet steam at the same temperature ($p = 69{\cdot}21$ lbs. per square inch, 6·168 cubic feet per pound) with 20 per cent. water in it. Find its intrinsic energy and its entropy.

(B. of E., Advanced, 1899.)

Solution.—See Ex. 7, Chapter III.

3. A pound of water at 0° C. is heated as water to 145° C. and then converted into wet steam at the same temperature with 15 per cent. of wetness (p is 60·4 lbs. per square inch, u is 7·009 cubic feet per pound); find its intrinsic energy and its entropy in excess of what they were at 0° C.

(B. of E., Advanced, 1900.)

Solution.—See Ex. 7, Chapter III.

4. Find the entropy added to 1 lb. of water at 181° C. in forming 1 lb. of wet steam at 181° C. if $\frac{9}{10}$ of it is steam and $\frac{1}{10}$ water.

Sketch the appearance of a water-steam ϕ-θ diagram, and show how it informs us about liquefaction during adiabatic expansion.

(B. of E., Advanced, 1901.)

Solution.—See Ex. 8, Chapter III.

5. The entropy of 1 lb. of water for the absolute Centigrade temperature t is

$$\phi = \log_e \frac{t}{273}.$$

Calculate this for two values of the temperature, say 70° C. and 170° C.

It is, of course, 0 at 0° C. Plot the ϕ-θ diagram for water. State exactly how much heat is represented by 1 square inch of your diagram.

(B. of E., Advanced, 1903.)

Solution.—See Ex. 9, Chapter III.

6. The entropies of a pound of water and dry saturated steam at 150° C. are 0·442 and 1·623 respectively. The volume of the dry saturated steam would be 6·168 cubic feet. Now, there is a pound of wet steam at 150° C., whose entropy is 1·235. What is its dryness fraction? That is, how much of it is steam and how much water? What is its volume? Neglect the volume of the water. B. of E., Stage II., 1905.)

Solution.—In fig. 49, place point A on the water line opposite 150° C. Draw the horizontal line A B to the saturated steam line. Mark the point C on this line where the entropy is 1·235. Then the dryness fraction is A C divided by A B. The volume at C is $6{\cdot}168 \times \frac{AC}{AB}$ cubic feet.

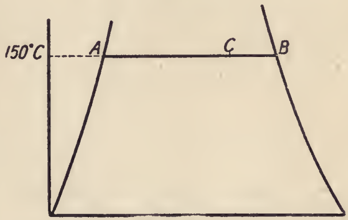

Fig. 49.

7. The entropy of 1 lb. of water at 0° C. is

$$\log_e \frac{273 + \theta}{273}.$$

What is this if θ is 160° C.? If this water is converted into dry saturated steam at 160° C., what is the additional entropy?

(B. of E., Stage II., 1907.)

Solution.—See Examples 3 and 6, Chapter III.

8. In a perfect steam engine 1 lb. of water at 100° C. is heated to 160° C. and then converted into dry steam; it expands adiabatically to 100° C., and is converted into water again at this temperature. Show this cycle on the θ-ϕ diagram with which you are supplied. The area represents in heat units or in work units the work done by a perfect steam engine working between these temperatures. How much is it?

(B. of E., Stage II., 1910.)

Solution.—This is the Rankine cycle. See Ex. 21, Chapter VI.

9. On the θ-ϕ diagram mark with letters A, B, C the points which show the following states of 1 lb. of water-stuff:—

A at 190° C. and 95 per cent. dry.
B at 100 lbs. per square inch and 60 per cent. dry.
C at 50 lbs. per square inch pressure and superheated to 210° C.

If a pound of water-stuff receives 3 Centigrade heat units, its temperature remaining constant at 150° C., what is its gain of entropy?

(B. of E., Stage II., 1910.)

Solution.—See Examples in Chapters III. and IV.

For last portion—

$$\text{Entropy} = \frac{\text{heat received}}{\text{absolute temperature}} = \frac{3}{150 + 273} \text{ ranks}$$
$$= \cdot 0071 \text{ ranks.}$$

10. On the temperature-entropy diagram which is given you, mark the point A, which shows the state of 1 lb. of steam whose pressure is 100 lbs. per square inch, and which is superheated to 210° C. If it expands adiabatically to 20 lbs. per square inch, how much of it is steam, how much water? (B. of E., Stage II., 1911.)

Solution.—See Examples in Chapter IV.

11. Dry steam becomes wet in adiabatic expansion, and very wet steam becomes drier; use your temperature-entropy diagram to find what sort of steam exhibits very little condensation or vaporisation during adiabatic expansion. (B. of E., Stage II., 1911.)

Solution.—See Examples in Chapter III.

12. How many thermal units are required for the formation from water at 30° C. of one pound of steam at a pressure of 150 pounds per square inch in each of the following states:—

(*a*) Dry and saturated?
(*b*) Wet with a dryness fraction of 0·8?
(*c*) Superheated 30° C. above the temperature of evaporation at the constant pressure of formation?

Mark the points in the temperature-entropy diagram corresponding to each state respectively by the letters A, B, C, and distinguish the areas concerned by cross-hatching. (B. of E., Stage II., 1912.)

Solution.—See Chapters III. and IV.

13. Show by means of the θ-ϕ diagram how we easily find the condensation which occurs during adiabatic expansion for any amount of wetness of steam. (B. of E., Hons., 1897.)

Solution.—See Examples 2 and 4, Chapter III.

14. One pound of stuff consisting of 0·6 lb. of steam and 0·4 lb. of water expands adiabatically from 311° F. to 230° F. What is the weight of water at the end? What are the volumes v of steam at the beginning and at the end? Neglect volumes of water.

Temperature.	p Pressure.	Volume of 1 Lb. of Steam.	ϕ for 1 Lb. of Water.	ϕ for 1 Lb. of Steam.
311° F.	79·03	5·433	·451	1·612
230° F.	20·80	19·03	·339	1·716

(B. of E., Hons., Part I., 1898.)

Solution.—See Ex. 19, Chapter V.

15. Sketch an indicator diagram such as might be expected from a non-condensing engine with a common slide valve. How do we show on it the dryness of the steam during expansion? What information must be given to enable us to do this? Show how we convert the expansion part into a θ-ϕ diagram. (B. of E., Hons., Part I., 1899.)

Solution.—See Ex. 25, Chapter VII.

16. A perfect steam engine, Rankine cycle; given the higher and lower temperatures and initial wetness or amount of superheating. Using a θ-ϕ diagram, show how you would find the work done per pound of stuff. If the stuff is released before the end of the expansion, show the amount of lessening of work done. (B. of E., Hons., Part I., 1899.)

Solution.—See Ex. 22, Chapter VI.

17. Given the following numbers, draw a θ-ϕ diagram:—

Temperature.	100° C.	130° C.	160° C.
Entropy of 1 lb. water, .	·314	·393	·466
Entropy of 1 lb. steam, .	1·748	1·667	1·604

Steam is 90 per cent. dry at 160° C.; find its dryness as it expands adiabatically, at 130° C. and at 100° C.

(B. of E., Hons., Part I., 1901.)

Solution.—See Ex. 2, Chapter III.

18. A non-condensing engine uses 4,000 lbs. of dry saturated steam per hour at 160° C.; feed water at 20° C. The I.H.P. is 210; what is the efficiency? How much work is done per pound of steam? If a perfect steam engine works on the Rankine cycle between 100° C. and 160° C., what work is hypothetically possible per pound of steam? Use the table of numbers given in the previous question.

(B. of E., Hons., Part I., 1901.)

Solution.—See Examples 21 and 22, Chapter VI.

19. Find the heat given to 1 lb. of feed water at 40° C. to convert it into wet steam (15 per cent. water) at 170° C. If 25 lbs. of this wet steam reaches the cylinder per horse-power-hour, what percentage of heat leaves with the exhaust or is radiated from the cylinder?

Taking the figures given in the following table, draw a θ-ϕ diagram :—

Temperature.	170° C.	100° C.	40° C.
Entropy of 1 lb. water, .	0·490	0·314	0·137
Entropy of 1 lb. steam, .	1·585	1·748	1·982

State in heat units and in foot-pounds the energy that is represented to scale by 1 square inch of your figure. Find the work that would be done per pound of this wet steam in a perfect steam engine (Rankine cycle) working between these temperatures of 170° C. and 40° C. What is the efficiency ratio of the engine as compared with the perfect steam engine. Answers must be correct to 1 per cent. The examiners do not want to be told how calculations are made on the θ-ϕ diagram. Candidates must really make the calculations correctly. Also, calculation by a formula is not what is here wanted. (B. of E., Hons., Part I., 1902.)

Solution.—The first part of the question must be calculated in the ordinary way. The area under the water line and horizontal line to steam line, down to absolute zero, is the total heat required to form the steam. The second part of the question is solved by the θ-ϕ chart. See Ex. 21, Chapter VI., and the work of Chapter VI. generally.

20. Feed water 25° C.; steam 10 per cent. wet—that is, there is 0·1 lb. of water to 0·9 lb. of steam at 170° C. If 25 lbs. of this wet steam enter the cylinder per indicated horse-power-hour, how much of the heat passes to the exhaust? If the stuff leaves the cylinder as saturated steam and water at 105° C., what is its wetness? Neglect radiation or other loss of heat by the cylinder. (B. of E., Hons., Part I., 1903.)

Solution.—As the expansion is adiabatic, the wetness can be found from the chart in the usual way. See Chapter III.

21. On an indicator diagram there is a point on the expansion curve where the absolute pressure is 85 lbs. per square inch. If at this point there is a weight of water in the cylinder just equal to the weight of the steam, show on a θ-ϕ diagram how the stuff changes during expansion. What is the total heat gained or lost by it from $p = 85$ to $p = 25$?

You are given the following information :—

p, Lbs. per Sq. Inch.	$\theta°$ C.	ϕ for 1 Lb. of Water.	ϕ for 1 Lb. of Steam.	u, Cubic Feet (vol. of 1 Lb. of Steam.)
85	158	0·461	1·608	5·07
50	138·2	0·415	1·649	8·34
25	115·6	0·356	1·704	15·99

(B of E., Stage III., 1904.)

Solution.—See Ex. 26, Chapter VII.

22. Given the following information, draw a θ-ϕ diagram. A pound of water-steam at 160° C. expands adiabatically to 115° C. If 90 per cent. of it is steam at the beginning, how much of it is steam at the end? If only 30 per cent. is steam at the beginning, how much of it is steam at the end?

$\theta°$ C.	ϕ for 1 Lb. of Water.	ϕ for 1 Lb. of Steam.	U, Cubic Feet of Steam per Lb.	p, Pressure in Lbs. per Sq. In.
160	·466	1·604	4·817	89·86
115	·354	1·705	16·32	25·54

What is the actual volume v at the beginning and end in both cases, neglecting the volume of the water part? Assume that in each case there is an adiabatic law like $p\,v^n$ constant, and find n.

(B. of E., Stage III., 1904.)

Solution.—See Examples 19 and 20, Chapter V.

23. Using the following information, draw a θ-ϕ diagram for water and steam:—

$\theta°$ C.	p.	Entropy of 1 Lb. of Water.	Entropy of 1 Lb. of Steam.	Vol. in Cub. Ft. of 1 Lb. of Steam.
100	14·7	0·313	1·749	26·43
150	69·2	0·441	1·623	6·168
200	226	0·556	1·536	2·031

State the amount of heat that is represented by 1 square inch of your diagram. In the expansion of 1 lb. of stuff, the following pressures and volumes are given :—

p, . . .	226	69·2	14·7
v, . . .	1·70	5·56	26·1

Mark these three points on the θ-ϕ diagram. How much heat is given to the stuff during this expansion ?

(B. of E., Stage III., 1905.)

Solution.—See Examples 19 and 20, Chapter V.

24. In the following table, u is the volume in cubic feet of 1 lb. of dry saturated steam. v is the actual volume of 1 lb. of wet steam (neglect the small volume of the water). What is the dryness fraction in each case ? In the table we give ϕ_w, the entropy of 1 lb. of water, and ϕ_s the entropy of 1 lb. of dry saturated steam. Make a θ-ϕ diagram. Mark the three points where the volume is 3·2 cubic feet, and draw a curve through them.

θ	u	ϕ_w	ϕ_s	v
175	3·419	0·500	1·575	3·2
150	6·168	0·441	1·623	3·2
120	14·04	0·366	1·692	3·2

(B. of E., Stage III., 1906.)

Solution.—See Examples 19 and 20, Chapter V.

25. What is the entropy of a pound of water-steam at θ_1° C., dryness fraction 0·9 ? If it expands adiabatically to θ_2° C., what is its dryness ? Take as an example $\theta_1 = 160$, $\theta_2 = 40$. If this steam, supplied at θ_1 to a turbine and exhausting at θ_2 is found to be of dryness 0·95, the work done per pound of steam is less than what the Rankine cycle gives ; how much less ? Assume no radiation of heat from the turbine.

(B. of E., Stage III., 1907.)

Solution.—See Ex. 24, Chapter VI.

26. A quantity of steam expands adiabatically from one of the following pressures to the other. If θ° C. is the temperature, v the volume in cubic feet, u the volume in cubic feet of 1 lb. of dry saturated steam at each of these temperatures ; what are the weights of water and steam in each case ? ϕ_w and ϕ_s are the entropies of 1 lb. of water and 1 lb. of steam.

p	$\theta°$ C.	v	u	ϕ_w	ϕ_s
101·9	165	8·58	4·290	·476	1·593
17·53	105	43·90	22·40	·326	1·734

(B. of E., Stage III., 1908.)

Solution.—See Examples 19 and 20, Chapter V.

27. 18 lbs. of steam per hour per I.H.P. enters a cylinder at 175° C., being 95 per cent. dry; assuming no heat to be given out from the cylinder, how much heat is in the exhaust steam at 105° C.? What is its dryness? If it had expanded to 105° C. adiabatically, what would have been its dryness? You are given the following values:—

Temperature.	175° C.	105° C.
Entropy of 1 lb. water, . . .	·500	·326
Entropy of 1 lb. steam, . . .	1·575	1·734

(B. of E., Stage III., 1908.)

Solution.—See Examples in Chapter III.

28. It has been said that there is a great gain of efficiency in using superheated steam, for the reason that the efficiency of a heat engine depends upon the highest and lowest temperatures. Show that this is of small importance here. In what way does the superheating of steam lead to economy?

(B. of E., Stage III., 1909.)

Solution.—See Ex. 16, Chapter IV., and Chapter IV. generally.

29. On the θ-ϕ chart with which you are supplied mark a line A B showing the adiabatic expansion of 1 lb. of steam at 150 lbs. per square inch, and 95 per cent. dry, to 10 lbs. per square inch. Give three readings of p and v during this expansion. Is there approximately a law of the form $p\,v^n$ constant, and, if so, what is n? (B. of E., Stage III., 1910.)

Solution.—See Examples in Chapter III., and examples on volumes in Chapter V.

30. By means of the θ-ϕ diagram, find how much heat must be given to 1 lb. of dry steam at 130 lbs. per square inch, in expanding to 10 lbs. per square inch, keeping just dry. (B. of E., Stage III., 1910.)

Solution.—In fig. 50, the sectional area is the heat that must be given the steam in order that it may remain just dry during the expansion.

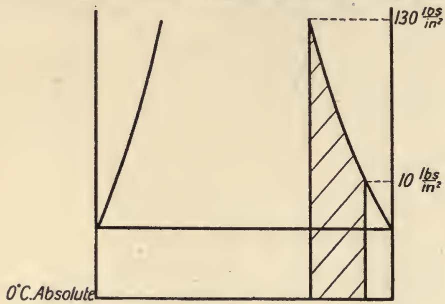

Fig. 50.

31. Mark the following points on your temperature-entropy diagram :—

A. One pound of steam at 100 lbs. per square inch superheated to 210° C. If this expands adiabatically, at what pressure is it just dry steam with no superheat ?

B. One pound of steam at 200 lbs. per square inch, 30 per cent. dry. Let this expand to 20 lbs. per square inch adiabatically ; what is now its dryness ? (B. of E., Stage III., 1911.)

Solution.—See Chapters III. and IV.

32. Show on your temperature-entropy diagram the Rankine cycle for a pound of dry steam at 150 lbs. per square inch if the feed water is at 40° C. If the expansion ceases at 50 lbs. per square inch, but the exhaust is still 40° C., what is the cycle ? What is the percentage loss of work due to the incompleteness of the expansion ? (B. of E., Stage III., 1911.)

Solution.—See Chapter VI.

33. Why is it economical to use superheated steam ? Show from a sketch on the temperature-entropy diagram that the thermo-dynamic advantage of superheating is not of much importance. (B. of E., Stage III., 1911.)

Solution.—See Ex. 16, Chapter IV.

34. The indicator diagram supplied to you was taken from an engine with a cylinder 50 inches diameter and 5 feet stroke. The clearance is 6 per cent. of the effective volume. Add the axes of pressure and volume and, with the aid of the temperature-entropy diagram, find the weight of steam in the cylinder at the point marked P on the expansion curve ?

(B. of E., Stage III., 1912.)

Solution.—See Chapter VII

35. Mark with a letter A the point of the temperature-entropy diagram which shows one lb. of steam at 150 lbs. per square inch pressure absolute superheated to 210° C. Find from the diagram the dryness of the steam after it expands adiabatically from the state A to a back pressure of one lb. per square inch absolute, and find also the volume of steam in the final state. (B. of E., Stage III., 1912.)

Solution.—See Chapters IV. and V.

36. Cross-hatch the area on the temperature-entropy diagram which represents the work done per lb. of steam by the Rankine engine, working between the initial pressure of 150 lbs. per square inch absolute, and the final pressure of 20 lbs. per square inch absolute. Show the additional work which can be obtained if the steam is superheated to 210° C., before the process of adiabatic expansion begins. (B. of E., Stage III., 1912.)

Solution.—See Chapter VI.

37. Assuming a cylinder to be perfectly non-conducting, show that, knowing exactly a very short part of the expansion curve, and knowing the initial pressure, we can calculate the water in the cylinder before and after admission. (B. of E., Hons., Part II., 1898.)

*Example A.**—Fig. 51 shows a diagram taken from an ordinary steam engine. At G, presumably the end of admission, the volume is 1·4 cubic feet, pressure 51·9 lbs. per square inch (or 283·2° F.), $u_1 = 8{\cdot}071$, latent heat $F = 914{\cdot}5$, and thus the indicated steam is $i = 0{\cdot}173$ lb. At F the volume is 4·323 cubic feet, pressure 21·69, and the steam present weighs 0·235 lb.

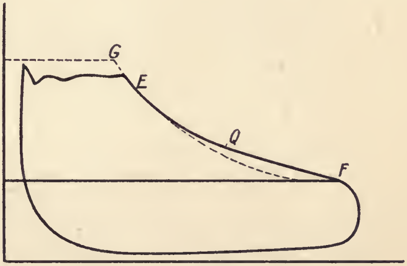

Fig. 51.

Assume that the metal of the cylinder is non-conducting, that we have z lbs. of *water* present before admission, that i lb. is the indicated steam at G, that the entering steam which has condensed is $y\,i$; neglect the steam present at the end of the cushioning; assume that all the water stuff present is everywhere of the same temperature at any instant, and, therefore, that the expansion is adiabatic, it is evident that we have the means of calculating y and z.

The latent heat $l\,y\,i$ of $y\,i$ lb. of steam heats z lb. of water from the back pressure temperature ($p_3 = 3{\cdot}4$ or $t_3 = 146{\cdot}3°$ F. say) to 283°, so that

$$914{\cdot}5\,y \times 0{\cdot}173 = 137z,$$

$$\text{or } y = 0{\cdot}866z. \quad . \quad . \quad . \quad . \quad (1)$$

* This and the following examples, taken by permission from Professor Perry's *The Steam Engine and Gas and Oil Engines*, will serve to illustrate this and similar cases.

Let the whole water stuff present,

$$\cdot173 + z + \cdot173\, y = w.$$

Again, if the adiabatic expansion represented by G E Q F on the indicator diagram is represented by G′ F′ on the θ-ϕ diagram (fig. 52), then

$$\frac{g\,G''}{g\,G'} = \frac{w}{\cdot173},$$

$$\frac{f\,F''}{f\,F'} = \frac{w}{\cdot235},$$

so that, $$\frac{g\,G''}{g\,G'} \cdot \frac{f\,F'}{f\,F''} = \frac{\cdot235}{\cdot173}.$$

Hence, $$\frac{g\,G'}{f\,F'} = \frac{g\,G''}{f\,F''} \cdot \frac{\cdot173}{\cdot235}.$$

Now, $$g\,G'' = \frac{\text{latent heat } 914\cdot5}{\text{absolute temperature } 744\cdot2} = 1\cdot229,$$

$$f\,F'' = \frac{952}{693} = 1\cdot374,$$

$$\frac{g\,G''}{f\,F'} = \frac{\cdot173}{\cdot235} \times \frac{1\cdot229}{1\cdot374} = \cdot660.$$

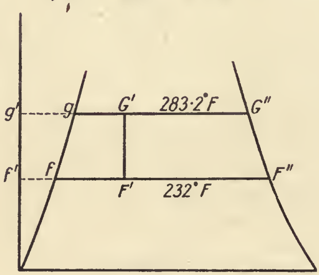

Fig. 52.

We have now only to find the two points G′ and F′, so that they shall be in the same vertical, and the distances in this ratio. We can do this by trial, when

$$\frac{g\,G''}{g\,G'} = 8\cdot72 = \frac{w}{\cdot173}.$$

Hence, $w = 1\cdot508$.

Or, without actual trial on a diagram; from the tables $g'\,g$ for 283·2° F. is ·415, $f'\,f$ for 232° F. is ·342.

Let $g'\,G' = f'\,F' = x$, then,

$$\frac{x - \cdot415}{x - \cdot342} = \cdot660, \text{ whence } x = \cdot556,$$

$$g\,G' = \cdot556 - \cdot415 = \cdot141,$$

or, $$G'\,G'' = 1\cdot088.$$

So that there is 7·71 times as much water present as steam at the point of cut-off.

In fact,
$$w = \frac{1{\cdot}229}{{\cdot}141} \times {\cdot}173 = 1{\cdot}508$$
$$= {\cdot}173 + z + {\cdot}173\, y$$
$$= {\cdot}173 + z + ({\cdot}173 \times {\cdot}866\, z),$$
whence
$$z = 1{\cdot}16,\ y = 1{\cdot}005, \text{ or } y\,i = {\cdot}174.$$

That is, the whole $w = 1{\cdot}508$ is made up of (at cut-off) indicated 0·173, condensed 0·174, water already there 1·16; which is a very striking sort of result.

Example B.—Given v_1, the volume in cubic feet of steam at the end of admission, the indicated steam is $i_1 = \frac{v_1}{u_1}$ lbs.

Suppose x lb. (we called it $y\,i$ in Example A) to have condensed during admission, its latent heat $l_1\,x$ has been given to z lb. already in the cylinder, to heat it from t_3 to t_1, so that

$$z\,(t_1 - t_3) = l_1\,x. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Let p_2 and v_2 be the pressure and volume at any other part of the expansion curve, i_2 being the weight of indicated steam there; if we assume that the metal of the cylinder is perfectly non-conducting, we can calculate x and z.

Let the entropy of 1 lb. of water be ϕ, and let $i_1 + x + z = w$.

Then,
$$\frac{w}{i_1} = \frac{g\,G''}{g\,G'}, \text{ and } \frac{w}{i_2} = \frac{f\,F''}{f\,F'},$$

and we find that, calling $\frac{l_1}{t_1}\ a_1$, and $\frac{l_2}{t_2}\ a_2$:—

$$w = \frac{a_2\,i_2 - a_1\,i_1}{\phi_1 - \phi_2} = i_1 + x + z. \quad . \quad . \quad . \quad . \quad (2$$

(1) and (2) are equations connecting x and z; we find the unknowns to be—

$$z = \left(\frac{a_2\,i_2 - a_1\,i_1}{\phi_1 - \phi_2} - i_1\right) \div \left\{1 + \frac{1}{a_1}\left(1 - \frac{t_3}{t_1}\right)\right\},$$

and
$$x = z\left(1 - \frac{t_3}{t_1}\right) \div a_1.$$

38. Rankine cycle, perfect steam engine, with dry steam at t_1, expanded adiabatically to t_3; find a formula for the work done per pound of steam. How do we find the answer graphically?

(B. of E., Hons., Part II., 1899.)

Solution.—See Ex. 21, Chapter VI.

39. Find the entropy of 1 lb. of wet steam, 20 per cent. of which is water at 160° C. (89·86 lbs. per square inch). Dry steam at this temperature measures 4·827 cubic feet to the pound. If at 100° C. ($p = 14{\cdot}7$ lbs. per

square inch, $u = 26{\cdot}43$ cubic feet per pound) it has the same entropy, how much of it is water? What are the volumes in these two cases, neglecting the volume of the water? If $p\ v^s$ is constant during adiabatic expansion, in this case find s.

(B. of E., Hons., Part II., 1899.)

Solution.—See Examples 19 and 20, Chapter V.

40. Assuming a cylinder to be non-conducting, show that, knowing exactly a very short part of the expansion curve, and knowing the initial pressure, we can calculate the water in the cylinder before and after admission. (B. of E., Hons., Part II., 1900.)

Solution.—See Examples A. and B., pp. 63 and 65.

41. Sketch the entropy diagram for steam at 190° C. ($p = 182{\cdot}4$ lbs. per square inch) superheated 50° C. above its temperature of production, expanded adiabatically to 40° C. and condensed. Find the work done per pound of steam. What is the state of steam as to dryness at the end of the expansion? (B. of E., Hons., Part II., 1900.)

Solution.—See Ex. 16, Chapter IV.

42. Assume a cylinder to be perfectly non-conducting and the water and steam all at the same temperature, the initial temperature being 145° C. and the pressure 60·40 lbs. per square inch. At two points on the expansion curve we find the following values of v and p:—

v, Cubic Feet.	p, Lbs. per Sq. Inch.	Temperature.	Vol. of 1 Lb. of Dry Sat. Steam.	Entropy of 1 Lb. of Water.	Entropy of 1 Lb. of Steam.
..	60·4	145° C.	7·01	0·430	1·634
2·8	52	139·4° C.	8·1	0·4174	1·6454
6·1	23	117·1° C.	17·4	0·3595	1·6695

We have found the temperature, etc., of steam corresponding to the observed pressures, and here given them. Find how much water was in the cylinder (the temperature then being 63° C.) just before admission, and how much steam was condensed on admission. Neglect the amount of steam present before admission.

(B. of E., Hons., Part II., 1901.)

Solution.—See Examples A and B, pp. 63 and 65.

43. Taking the following figures, draw a θ-ϕ diagram. State in heat units and in foot-pounds the energy that is represented to scale by 1 square inch of your figure. Find the work that would be done by 1 lb. of steam 90 per cent. dry at 160° C. on the Rankine cycle, the lower temperature being 100° C. Your answer must be correct to 3 per cent. :—

Temperature.	160° C.	130° C.	100° C.
Entropy of 1 lb. water	0·465	0·391	0·313
Entropy of 1 lb. steam	1·603	1·668	1·749

Suppose release to take place before 100° C. is reached in the expansion, what assumption is made to enable us to represent release on the θ-ϕ diagram? (B. of E., Hons., Part II.,1903.)

Solution.—See Ex. 23, Chapter VI.

44. Assuming a cylinder to be non-conducting, that there is no leakage past piston or valves, and that all the stuff, water, and steam are at the same temperature, show that, knowing exactly a very short part of the expansion curve and the initial pressure, we can calculate the water in the cylinder before and after admission.

(B. of E., Hons., Part II., 1903.)

Solution.—See Examples A and B, pp. 63 and 65.

45. Given the following information, draw a θ-ϕ diagram. A quantity of water-steam, whose weight is unknown, has the volume 6·16 cubic feet at 160° C. It expands adiabatically to 115° C., and then its volume is 26·27 cubic feet; neglect the volume of the water part:—

θ° C.	ϕ of 1 Lb. of Water.	ϕ of 1 Lb. of Steam.	*u*, Cubic Feet of Steam per Lb.
160	0·466	1·604	4·827
115	0·354	1·705	16·32

What is the weight of stuff with which we are dealing, and how much of it is steam and how much of it water at the beginning and at the end?

(B. of E., Hons., 1904.)

Solution.—The volume 4·827 cubic feet is represented on the entropy scale by (1·604 − 0·466) = 1·138.

Therefore, 6·16 cubic feet will have a corresponding length of

$$\frac{1·138 \times 6·16}{4·827} = 1·452.$$

The total length from the line of temperature will be 1·452 + 0·466 = 1·918.

In the same way 26·27 cubic feet are given by

$$(1·705 - 0·354)\frac{26·27}{16·32} = 2·174,$$

and the total length from the temperature ordinate is

$$2{\cdot}174 + 0{\cdot}354 = 2{\cdot}528.$$

Now, as the expansion is adiabatic, we must add water, in order to make the expansion line on the chart vertical.

The additional water necessary to increase the entropy from 1·918 to $2{\cdot}528 = \frac{2{\cdot}528 - 1{\cdot}918}{0{\cdot}466 - 0{\cdot}354}$, because each pound increases the entropy by $0{\cdot}466 - 0{\cdot}354$.

Therefore, total water and steam present at the beginning is

$$\frac{0{\cdot}610}{0{\cdot}112} + 1 = 6{\cdot}446 \text{ lbs.},$$

and of this, $\frac{6{\cdot}16}{4{\cdot}827} = 1{\cdot}276$ lbs. are steam, so that dryness at the beginning $= \frac{1{\cdot}276}{6{\cdot}446} = 0{\cdot}192$.

The steam present after the adiabatic expansion $= \frac{26{\cdot}27}{16{\cdot}32} = 1{\cdot}61$ lbs., and dryness after expansion $= \frac{1{\cdot}61}{6{\cdot}446} = 0{\cdot}25$.

46. What information is necessary if we wish to convert an indicator diagram into a θ-ϕ diagram? How is it done? What assumptions do we make? Although these assumptions are untrue, show that we can use the p-v, or the corresponding θ-ϕ diagram in obtaining practical knowledge. (B. of E., Hons., 1906.)

Solution.—See Chapter VII.

47. Assume that in the cylinder the water is at the same temperature as the steam during admission and expansion. Assume the expansion to be adiabatic, the cut-off being quick. The following measurements of p and v are made, 163·6 lbs. per square inch being the pressure of admission :—

$\theta°$ C.	p.	v.	u.	ϕ_w	ϕ_s
185	163·3	1·433	2·756	·524	1·560
165	101·9	2·313	4·29	·478	1·595

v is in cubic feet; $\theta°$ C. is the temperature; u is in cubic feet of each kind of steam per pound; ϕ_w is the entropy of 1 lb. of water, and ϕ_s the entropy of 1 lb. of steam of this temperature and pressure. How much water is there in the cylinder at each of the two points? How much water was in the cylinder before admission? Assume the temperature of the water present before admission to be 80° C., and neglect the weight of the steam present before admission. (B. of E., Hons., 1907.)

Solution.—See Examples A and B.

48. Taking the following values, draw a θ-ϕ diagram. State in heat units and in foot-pounds the energy represented by 1 square inch of your figure. Find the work per pound of dry steam at 165° C. on the Rankine cycle, the lower temperature being 100° C. What is the dryness of the exhaust steam? The exhaust steam of a real engine working between these temperatures is found to be at 100° C. and 95 per cent. dry; what is the work actually done per pound of steam? Assume no loss of heat by radiation from the cylinder:—

Temperature.	165° C.	100° C.
Entropy of 1 lb. water, . . .	·476	·313
Entropy of 1 lb. steam, . . .	1·593	1·749

(B. of E., Hons., 1908.)

Solution.—See Ex. 24, Chapter VI.

49. Using the θ-ϕ diagram with which you are supplied, what work is done per pound of dry steam on the Rankine cycle between 180° C. and 100° C.? What is the dryness of the exhaust steam? The exhaust steam of an engine working between these temperatures is found to be 97 per cent. dry. What is the work actually done per pound of steam? The answer is represented on the diagram by the difference between two areas: what are they? Assume no heat to be lost by radiation from the engine.

(B. of E., Hons., 1910.)

Solution.—See Ex. 24, Chapter VI.

50. If 20 lbs. of steam, which is 95 per cent. dry at 180° C., enter a cylinder per hour per indicated horse-power, and if we assume that no heat is given out from the cylinder, what is the dryness of the exhaust steam at 55° C.? If it had expanded adiabatically to 55° C., what would have been its dryness? Use your temperature-entropy diagram. On such diagram the work done per pound of steam is the difference between two areas. What are they? (B. of E., Hons., 1911.)

Solution.—See Chapter III., and Ex. 24, Chapter VI.

51. Mark the point on your temperature-entropy diagram where 1 lb of steam at 160° C. is 80 per cent. dry. What is its volume? Neglect the volume of the water. (B. of E., Hons., 1911.)

Solution.—See Chapter V.

Some Examples from the Examination Papers of the Institute of Civil Engineers.

52. What percentage of steam initially containing 10 per cent. of moisture will be liquefied during adiabatic expansion from 307° F. to 120° F.?

(I.C.E., 1903.)

Solution.—See Chapter III.

53. Explain what is meant by "entropy," and show how the change of state of a fluid consequent on the application of heat is represented graphically by a temperature-entropy diagram. A fluid receives heat (1) at a constant temperature of 300° F.; (2) as the temperature rises at a uniform rate from 300° F. to 500° F.; (3) at a constant temperature of 500° F.; the quantity of heat received in each stage being 1,000 thermal units. Calculate the change of entropy, and sketch the diagram.

(I.C.E., 1899.)

Solution.—See Chapter II.

54. Show how the heat supplied during the expansion of a mixture of steam and water is graphically represented on a temperature-entropy diagram. Show that if no heat is supplied to steam which is originally dry, it necessarily condenses during expansion, and exhibit graphically the heat necessary to prevent condensation. (I.C.E., 1898.)

Solution.—See Ex. 28, Chapter VIII.

55. Show how to construct the entropy diagram for steam, and state the use of the diagram.

Steam expands adiabatically from being initially wet. Find the change in the dryness fraction for a given range of temperature.

(I.C.E., 1904.)

Solution.—See Chapters II. and III.

56. If an indicator diagram of a steam engine cutting off at $\frac{3}{8}$-stroke, and working between a pressure of 100 lbs. and 30 lbs. absolute, were supplied to you, show fully how you would draw an entropy chart so as to find out the dryness fraction at the end of expansion. State what additional data would be required. (I.C.E., 1902.)

Solution.—See Chapter VII.

57. Define the term "thermal efficiency." Work out a formula for the thermal efficiency of the Carnot cycle for a heat engine, and for the Rankine (Clausius) cycle for a steam engine, or show the meaning graphically by means of the temperature-entropy chart. Why is the thermal efficiency of the Carnot cycle greater than that of the Rankine cycle?

(I.C.E., 1899.)

Solution.—See Chapter VI.

58. Find an expression for the Rankine or Clausius thermal efficiency for a steam engine receiving saturated steam at the stop-valve temperature T_a and exhausting at T_e. Sketch an entropy-temperature diagram, and show by means of areas how this efficiency may be graphically represented upon it. (I.C.E., 1901.)

Solution.—See Chapter VI.

59. Describe the Clausius-Rankine cycle commonly employed as a standard of efficiency in steam engines, and obtain an equation for the useful work done per pound of steam in an engine working with this cycle. (I.C.E., 1898.)

Solution.—See Chapter VI.

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